MARKOVIAN AND NON-MARKOVIAN EFFECTS ON QUANTUM SUPERPOSITIONS IN SINGLE AND DOUBLE QUANTUM DOTS

F.J.Rodríguez[1], L.Quiroga[1], and N.F.Johnson[2]
[1] Departamento de Física, Universidad de Los Andes, A.A. 4976
Bogotá D.C., Colombia
[2] Physics Department, Clarendon Laboratory, Oxford University
Oxford OX1 3PU, U.K.

RESUMEN

Consideramos dos tipos de nanoestructuras simples como son: un punto (SQD) y dos puntos cuánticos (DQD), los cuales están acoplados a un campo de luz clásico y a dos baños, uno de fotones y otro de fonones. Estos baños proveen las fuentes escenciales de dephasing puro y relajación de los estados en el punto cuántico. Mostramos resultados, en los cuales la función de correlación de segundo orden ($g^{(2)}$) de los fotones emitidos, muestran un fuerte grado de antibunching sobre escalas de tiempo en las cuales los efectos no-Markovianos son de gran importancia. Estas características son únicamente observables cuando el estado inicial de preparación del punto cuántico es un estado de superposición, mostrando de esta manera la altisima sensibilidad de la ($g^{(2)}$) al estado de preparación. Para el caso de un punto cuántico doble, mostramos que la interacción dipolar es la responsable de ver efectos de superfluorescencia a tiempos muy cortos, cuando el laser es apagado y los dos dots son diferentes.

ABSTRACT

We consider a laser driven nanostructure such a single (SQD) or double quantum dot (DQD) of any shape, which is coupled to photon and phonon baths. These baths provide the basic sources of pure dephasing and relaxation of QD states. We present results which demonstrate that the second order correlation function ($g^{(2)}$) of emitted photons shows enhanced antibunching features on a time scale where non-Markov signatures are indeed important for a SQD. This latter feature is only observable when the initial QD state is of a superposition kind thus providing a remarkable sensitivity to state preparation. For a DQD we show that the dipole-dipole interaction is the responsible for superfluorescence at very short times when the driven laser is turned offi for dinstiguishable coupled quantum dots.

Quantum dots (QDs) are solid-state nanostructures which are analogs of real atoms[1]; semiconductor QDs and macromolecules are seen as excellent candidates for performing quantum information processing tasks[2, 3]. Essential steps toward the implementation of standard quantum information schemes in such nanostructures, include: (1) the identification of the basic qubit (quantum bit), and (2) the application of one- and two-qubit quantum gates in order to generate quantum superpositions and entanglement. Reference [2] showed that (1) and (2) can be achieved with excitons generated using current techniques in ultrafast optical spectroscopy[4].

Theoretical Model

We calculate the density matrix for an exciton interacting with both a phonon bath and a photon bath and from it we obtain the necessary information for building the second order coherence function $g^{(2)}$. The reduced density matrix for both SQD ($N = 1$) or DQD ($N = 2$) is obtained from

$$i\frac{\partial\rho}{\partial t} = \sum_{i=1}^{N}\epsilon_i[S_i^z,\rho] + \sum_{i=1}^{N}\Omega_i[S_i^\dagger + S_i^-,\rho] + \sum_{i\neq j}^{N}\Omega_{i,j}[S_i^\dagger S_j^-,\rho] + i\hat{L}\rho \qquad (1)$$

where $\epsilon_i = \omega_i - \omega$ (ω_i the exciton energy of the $i-$th QD) is the detuning, ω the laser frequency, Ω_i the laser Rabi frequency, $\Omega_{i,j}$ the dipole-dipole interaction between two QDs and $\hat{L}$ denotes the Liouvillian superoperator which contains the coupling of the nanostructure to the photon/phonon baths. $\S_i^\dagger$ are the exciton rising operators. The time correlation measurement of the radiation field emitted by a driven localized system is written in terms of field system's operators assuming the photodetectors are far enough from the system to avoid local field fluctuations. It is associated to the interference of transition amplitudes of a quantum radiation field, i.e., is a statistical measurement. To obtain the time evolution of the the exciton rising operators, we apply $< S_i >= Tr(\rho S_i)$ where the trace is over the dots plus the laser mode subsystem. For the DQD the important time evolution equations are given by

$$\begin{aligned}
\frac{dS_1^\dagger}{dt} &= -(\gamma_1 + i\Delta_1)S_1^\dagger - (\Gamma_{12} - i\Omega_{12})S_2{}^\dagger + 2(\Gamma_{12} - i\Omega_{12})S_1^\dagger S_1^- S_2^\dagger \qquad (2)\\
&+ i\Omega_1 S_1^\dagger S_1^- - i\Omega_1\\
\frac{dS_1^\dagger S_1^-}{dt} &= -2\Gamma_1 S_1^\dagger S_1^- - (\Gamma_{12} - i\Omega_{12})S_1^\dagger S_2^- - (\Gamma_{12} - i\Omega_{12})S_2^\dagger S_1^- \qquad (3)\\
&- \frac{i\Omega_1}{2}(S_1^- - S_1^\dagger)\\
\frac{dS_1^\dagger S_2^-}{dt} &= -(\gamma - i\delta)S_1^\dagger S_2^- - (\Gamma_{12} - i\Omega_{12})S_2^\dagger S_2^- - (\Gamma_{12} + i\Omega_{12})S_1^\dagger S_1^- - \frac{i\Omega_1}{2}S_2^-\\
&+ i\Omega_1 S_1^\dagger S_1^- S_2^- - i\Omega_2 S_1^\dagger S_2^\dagger S_2^- + -\frac{i\Omega_2}{2}S_1^- + 4\Gamma_{12}S_1^\dagger S_1^- S_2^\dagger S_2^- \qquad (4)\\
\frac{dS_1^\dagger S_2^+}{dt} &= -(\gamma - i\Delta)S_1^\dagger S_2^- \frac{i\Omega_1}{2}S_2^\dagger - \frac{i\Omega_2}{2}S_1^\dagger + i\Omega_1 S_1^\dagger S_1^- S_2^\dagger + i\Omega_2 S_1^\dagger S_2^\dagger S_2^- \qquad (5)\\
\frac{dS_1^\dagger S_1^- S_2^-}{dt} &= -(2\gamma_1 + \gamma_2 - i\Delta_2)S_1^\dagger S_1^- S_2^- - (\Gamma_{12} - i\Omega_{12})S_2^\dagger S_2^- S_1^-\\
&- \frac{i\Omega_1}{2}(S_1^- S_2^- - S_1^\dagger S_2^\dagger) + \frac{i\Omega_2}{2}S_1^\dagger S_1^- - i\Omega_2 S_1^\dagger S_1^- S_2^\dagger S_2^- \qquad (6)\\
\frac{dS_1^\dagger S_1^- S_2^\dagger S_2^-}{dt} &= -2\gamma S_1^\dagger S_1^- S_2^\dagger S_2^- - \frac{i\Omega_1}{2}(S_2^\dagger S_2^- S_1^- - S_1^\dagger S_2^\dagger S_2^-) - \qquad (7)\\
&- \frac{i\Omega_2}{2}(S_1^\dagger S_1^- S_2^- - S_1^\dagger S_1^- S_2^\dagger) \qquad (8)
\end{aligned}$$

where $\gamma = \gamma_1 + \gamma_2$,$\delta = \Delta_1 - \Delta_2$,$\Delta = \Delta_1 - \Delta_2$ and the another relevant equations can be obtained by changing $1 \rightarrow 2$

Results

First, the results for a SQD, are shown in Figure 1, when non-Markovian effects are considered for different initial states. The second order time correlation function $g^{(2)}(T,\tau)$ $(T = (t_1 + t_2)/2, \tau = t_1 - t_2)$, where photons are detected at t_1 and t_2, shows an strong enhancement when the system is prepared in a quantum superposition of vacuum and a single exciton state (solid line in Fig. 1, full non-Markovian effects), γ_0 represents the Markovian decay constant, In the Markovian limit our results agree with previous results for low resolution times. Most importantly, we demonstrate that for a SQD prepared in a quantum superposition state a new photon correlation signature occurs. By contrast, this effect is absent for a SQD originally prepared in a statistical mixture of states (dashed lines in Fig. 1, Markovian limit). Current ultrafast spectroscopy should be able to detect this new effect. In the femto- to picosecond regime the usual Born-Markov approximation overestimates decay and dephasing effects. As it is well known the short time evolution of the reduced density matrix is overdamped by a

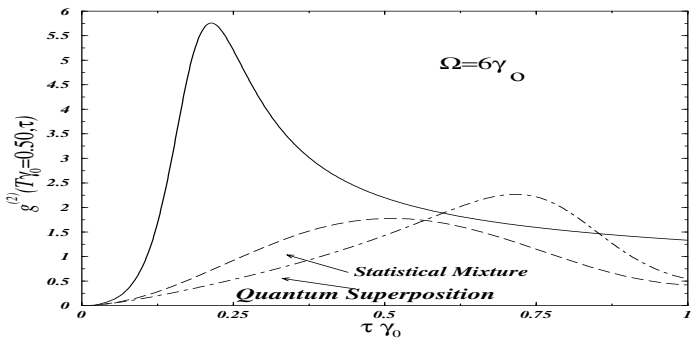

Figura 1: Second order correlation function for different initial states in a SQD

Markovian description[5]. A correct description of this short time dynamics imposes to resort to a non-Markovian equation of motion able to properly describe the quite short time regime where environmental effects are sensibly diminished. This fact has been identified as a result of reversible coupling with the environment. Next, the intensity $I(\tau)$ for an interdot spatial separation $r = \lambda/12$, $(\lambda = 2\pi c/w^+, w^+ = (\omega_1 + \omega_2/2,\ \Delta = \omega_1 - \omega_2)$ and different detunings are shown in Fig. 2a in the stationary limit. For coupled QDs a natural basis is the following: $|e_1e_2>, |g_1g_2>, 1/\sqrt{2}(|e_1g_2> \pm |g_1e_2>)$ (e_i denotes and excited state and g_i the ground state for the $i-th$ QD). For identical QDs, only the transition from the symmetric to the ground state is allowed. By contrast, for non-symmetrical QDs the antisymmetric state opens a new radiation channel and both, symmetric

and antisymmetric states can radiate, giving a superfluorescent effect in the emitted light. $g^{(2)}$ is showed in Fig. 2b when the laser is turned on and for different dipole-dipole interactions. In the strong field limit, and for long times, the photon correlation goes to 1. However, the dipole-dipole interaction does not affect $g^{(2)}$ at

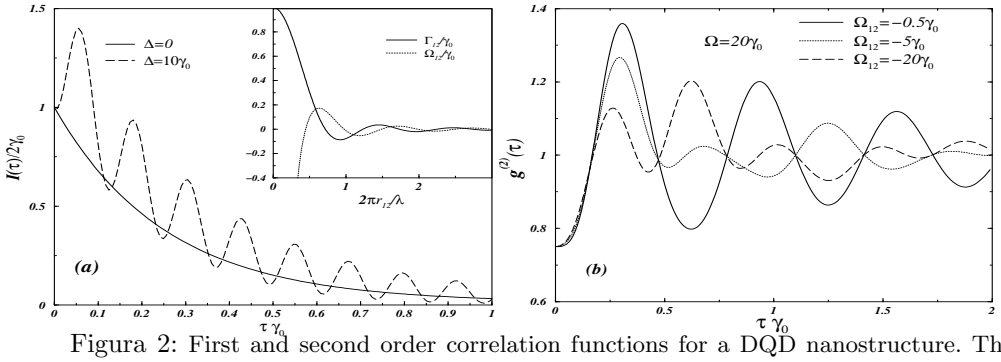

Figura 2: First and second order correlation functions for a DQD nanostructure. The inset shows the effective dipole-dipole intreraction as a function of the interdot separation, $\Gamma_{1,2}$ is the collective decay rate

$\tau = 0$ giving the same value, 0.75 for any dipole-dipole strength. As for a SQD a strong antibunching effect is still visible. The behaviour of photon correlation for DQD on a ultrafast scale will be published elsewhere.

In summary, we have shown a density matrix based approach to calculate photon correlations in an ultrafast scale where non-Markovian effects should be properly included. We propose an unambiguous signature for detecting quantum superpositions in a nanostructure.

This work has been partially supported by COLCIENCIAS project No.1204-05-11408 and Banco de la República (Colombia).

Referencias

[1] L. Jacak, P. Hawrylak and A. Wojs, *Quantum Dots*, (Springer, Berlin, 1998).

[2] L. Quiroga and N.F. Johnson, Phys. Rev. Lett. **83**, 2270 (1999); J.H. Reina, L. Quiroga, and N.F. Johnson, Phys. Rev.**A62**, 12305 (2000).

[3] D. Loss and D.P. DiVincenzo, Phys. Rev. **A57**, 120 (1998).

[4] N.H. Bonadeo et al., Phys. Rev. Lett. **81**, 2759 (1998).

[5] F.J.Rodríguez, L. Quiroga and N.F. Johnson, Phys. Rev. **B66**, 161302(RC) (2002).