DISTINGUISHING BETWEEN FRAUNHOFER AND FRESNEL DIFFRACTION

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ABSTRACT

Theoretical considerations, simulations and experiments have been carried out to distinguish between Fresnel and the Fraunhofer diffraction regarding the formation of interference patterns by a conventional Young's double-pinhole arrangement. We propose a criterion for distinguishing between Fresnel and Fraunhofer diffraction based on this number of Fresnel zones, which is applicable for diffracting apertures with any shape.

INTRODUCTION

FRESNEL AND FRAUNHOFER DIFFRACTION AND THE YOUNG'S EXPERIMENT

The intensity distribution recorded by a detector attached at the observation plane, which is z away from the aperture, will be given by [1]

$$I_{OP}(\vec{r}) = |u_{OP}(\vec{r})|^2 = \left(\frac{1}{\lambda z}\right)^2 \left| \int u_{AP}(\vec{\xi})\, t(\vec{\xi}) \exp\left[i \frac{k}{2z} |\vec{\xi}|^2 \right] \exp\left[-i \frac{k}{z} \vec{\xi} \cdot \vec{r} \right] d^2\xi \right|^2 \qquad [1]$$

where $u_{AP}(\vec{\xi})$ and $u_{OP}(\vec{r})$ denote the amplitude distribution of the light wave across the aperture and at the observation plane respectively, $t(\vec{\xi})$ is the aperture transmission and $k = \frac{2\pi}{\lambda}$ with λ the wavelength. The exponent $\frac{k}{2z}|\vec{\xi}|^2 = \frac{\pi|\vec{\xi}|^2}{\lambda z}$ in the integrand of eq.(1) is proportional to the ratio between the areas of a circle of radius $|\vec{\xi}|$ and of the first Fresnel's zone $\pi\lambda z$ respectively. This ratio approaches to the number of Fresnel's zones (N) [1] inscribed by the diffracting aperture from the intersection of the optical axis with the observation plane if the maximal circle inside the aperture is considered., i.e. $\pi N = \frac{\pi D^2}{\lambda z}$ with $D = |\vec{\xi}|_{MAX}$ If this number is enough small, the approach $\exp\left[i \frac{k}{2z} |\vec{\xi}|^2 \right] \approx 1$ will be valid, and the integral in eqs.(1) reduces to a Fourier transform. Under these conditions diffraction occurs in Fraunhofer domain, otherwise the influence of the Fresnel's zones on the diffraction phenomenon must be accounted, i.e. diffraction will occur in Fresnel domain [2,5,6]. According to eq.(1), the number of Fresnel's zones subtended by the diffracting aperture can be determined from intensity measurements at the observation plane, if the diffracting aperture is replaced by a mask with the adequate transmission. As a consequence, Fresnel and Fraunhofer diffraction can be distinguished by the same procedure. Such a mask consists of two identical circular pinholes on an opaque screen with one of them always centered on the optical axis. The pinhole separation will be variable but fulfills the condition $\left|\vec{b}\right| \leq D$.

Due to the diffraction patterns will remain unresolved for a range of enough small values of the separation vector length, in such a way that their absolute maximum essentially coincide at $\vec{r}_{MAX}^{(0)} = -\frac{\vec{b}}{2}$, the modulated interference fringes will exhibit an absolute maximum at this position, as reported by basic experiments on diffraction [1]. Under these conditions, we can express the intensity distribution recorded at the observation plane in a good approach as

$$I_{OP}(\vec{r}) = 4 I_0 \left(\frac{\pi a^2}{\lambda z}\right)^2 \left(\frac{2\, J_1\left(\frac{ka}{z}\left|\vec{r} + \frac{\vec{b}}{2}\right|\right)}{\frac{ka}{z}\left|\vec{r} + \frac{\vec{b}}{2}\right|} \right)^2 \cos^2\left[\frac{k\,\vec{b}}{2z} \cdot \left(\vec{r} + \frac{\vec{b}}{2}\right)\right] \qquad [2]$$

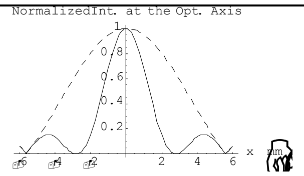
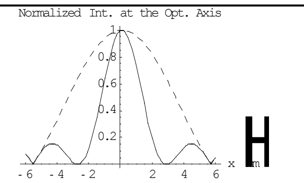
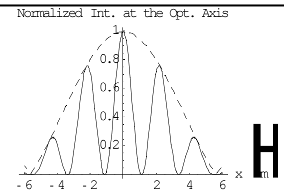
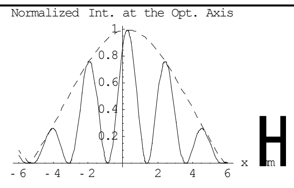
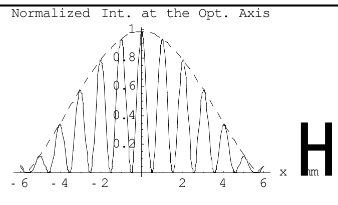
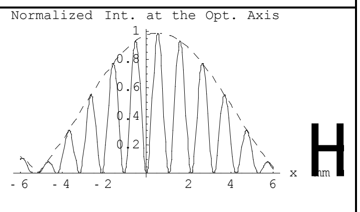
Where $J_n(y)$ denotes the Bessel's functions of first kind and *n*-th order. Note that the position of the zero-th order maximum of the interference fringes is due to an optical path difference $\delta = \frac{\left|\vec{b}\right|^2}{2\,z}$ in the set-up, which is corresponding to a phase difference $\phi = k\delta = \frac{k\left|\vec{b}\right|^2}{2\,z}$ between the two pinholes that appears as argument of the cosine function in eq.(2). This phase difference is proportional to the number of Fresnel zones inscribed by a circle of radius $\left|\vec{b}\right|$, located at the aperture plane and centered on the optical axis, i.e. $\phi = \pi N = \frac{\pi\left|\vec{b}\right|^2}{\lambda\,z}$.

In other words, the zero-th order maximum of the intensity distribution of the interference pattern in Fraunhofer domain should remain on the optical axis. If the pinhole separation increases in such a way that the Fresnel's phase must be considered, this maximum will be shifted appreciably [eq.(2)]. Consequently, the criterion to be proposed will be determined by the number of Fresnel's zones N_{FF} corresponding to the pinhole separation, which provides the small shift assumed as limit between the Fresnel and the Fraunhofer domains. Such a shift will be estimated by the decay of the intensity that is measured on the optical axis at the observation plane. Specifically, we regard a departure of 12 per cent of the recorded intensity value from the zero-th order maximum as the permissible limit. As a consequence

$$
\begin{aligned}
&N_{FF} \le 0.2232 \quad \text{for Fraunhofer diffraction}\\
&N_{FF} > 0.2232 \quad \text{for Fresnel diffraction}
\end{aligned}
\qquad [3]
$$

SIMULATION AND EXPERIMENTAL RESULTS

A set of two pinhole masks was made, with individual pinhole radius a= 0.10 ±0.01 mm and separations b from 0.20 ±0.01 mm to 1.50 ± 0.01 mm. The masks were illuminated with a plane wave from a He-Ne laser (λ = 632.8 nm) and the observation plane was arranged at 1.77 m from the mask. Each pinhole separation determines the radius of a diffracting aperture that subtends $N = \dfrac{b^2}{\lambda z}$ Fresnel's zones from the intersection of the optical axis with the observation plane. Table 1 shows some of the simulation results. It is apparent that the zero-th order maximum of the intensity distribution always appears at the optical axis (like in Fraunhofer diffraction) when the two pinholes are symmetrically attached with respect to the optical axis, no matter the value of N. It is due to the null value of the Fresnel's phase difference between the pinholes in such arrangement. The results on the third column of Table 1 show the lateral shifting of the intensity distributions at the observation plane if one of the pinholes remains fixed at the optical axis. As a consequence, the intensity value recorded on the optical axis at observation plane will vary as theoretically predicted [eq.(2)], with $\vec{r} = 0$.

Fresnel's zones	**Pinholes symmetrically distributed**	**Pinholes non-symmetrically distributed**
0.0357		
0.2232		
1.0000		

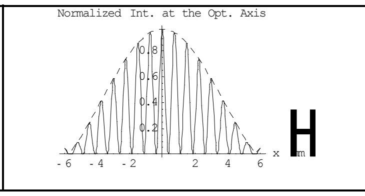
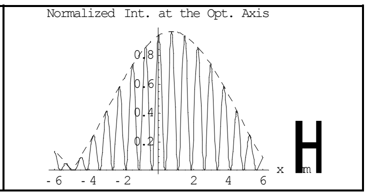
2.0000		

Table 1. Simulation result for the normalized intensity distribution at the observation plane.

The results of the experimental test of the proposed criterion for distinguishing between Fresnel and Fraunhofer domains are shown in Fig. 1. It shows the normalized intensity values at the recording point as a function of the pinhole separation. The circle dotted line denotes the simulation results whereas the square dotted line denotes the corresponding experimental results. A good agreement between these results is apparent.

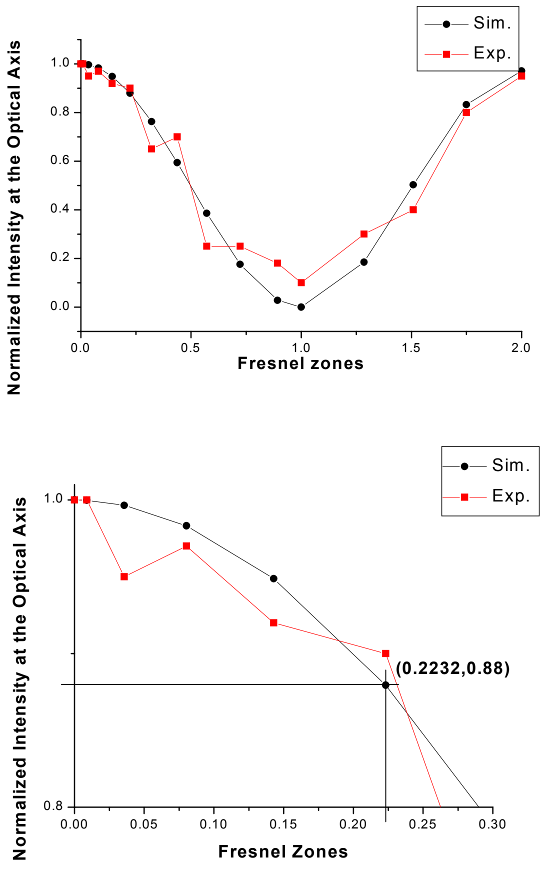


Fig. 1. Simulation and experimental results. a) The full range of Fresnel zones. b) A zoom of the region between 0 and 0.3 Fresnel zones.

CONCLUSION

Simulations and experiments based on the two pinhole Young's experiment provide an alternative analysis of Fresnel and Fraunhofer diffraction. Optical path differences in the set-up when one pinhole remains fixed at the optical axis fit the Fresnel's phase differences between the pinholes. For this reason it is possible to determine such phase differences by measuring intensity values at the observation plane.

We conclude that Fraunhofer diffraction will be obtained if the diffracting aperture subtends 0.2232 of the first Fresnel zone or less from the intersection between the optical axis and the observation plane. Otherwise Fresnel diffraction is obtained.

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