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APPLICATION OF THE LAWRENCE-DONIACH MODEL TO THE CALCULATION OF SCALING THERMODYNAMIC FUNCTIONS IN ANISOTROPIC HIGH TEMPERATURE SUPERCONDUCTORS

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ABSTRACT

It is considered a superconducting anisotropic system in an external magnetic field, applied perpendicular to the layered system. With the aid of the Lawrence-Doniach functional, in the lowest Landau level approximation, we obtain explicit expressions for the scaling functions of free energy, magnetization and specific heat of such system. It is shown that the interlayer coupling leads to a redefinition of the Ginzburg-Landau coefficient, α, depending on the phase difference of the order parameter between adjacent layers.

INTRODUCTION

During the last decade many efforts have been focused on the understanding of the critical properties of strongly type-II superconductors. In the frame of the Ginzburg-Landau (GL) theory, in a series of papers [1-3] it was developed a nonperturbative theory of critical behavior for anisotropic superconductors for both three- and two-dimensional systems. These results were generalized by Calero et al [4-6] to the case of an anisotropic three-dimensional superconductor with an arbitrary direction of the applied magnetic field. It was shown that for an arbitrary direction of the external magnetic field, the anisotropic superconductor displays one-dimensional fluctuations in a direction different from that of the magnetic field. It was also obtained a scaling function for the magnetization in the presence of magnetic fields parallel to the *ab* planes and the theoretical results were in good agreement with experimental data [7-9].

In the present communication we extend the method developed in [1-6] to the case of a layered system described by the Lawrence-Doniach model. For such model the mass anisotropy and the layered structure of the system can be accounted with more detail.

THEORETICAL FRAMEWORK

The system consists in an infinite array of superconducting layers located at $z = nd$ ($n = 0, \pm 1, \ldots$), where d is a constant denoting the minimum interlayer separation. An external magnetic field $\boldsymbol{B}$ is applied along the z axis. In order to avoid border effects we shall consider an infinite extension of the crystal in the *a* and *b* directions.

In order to describe such system we must consider the strong anisotropy of the system and the localization of the superconducting planes in the Ginzburg –Landau functional

$$F = \int_V \left\{ \frac{B^2}{8\pi} + \frac{\hbar^2}{4m} \left| \left(\nabla - \frac{2ie\boldsymbol{A}}{\hbar c} \right) \Psi \right|^2 + \alpha |\Psi|^2 + \frac{1}{2} \beta |\Psi|^4 \right\} dv \qquad (1)$$

Here F is the difference between the superconductor and the normal free energies, $\boldsymbol{A}$ is the vector potential ($\boldsymbol{B} = \nabla\times\boldsymbol{A}$), Ψ is the order parameter and, α, β are functions of temperature. The vector potential $\boldsymbol{A}$ is taken in the symmetric gauge $\boldsymbol{A} = \boldsymbol{B}\times\boldsymbol{r}/2$, where $\boldsymbol{r} = (x,y)$ is a two-dimensional (2D) vector. The mass anisotropy can be taken into account by introducing the inverse mass tensor $(1/m)_{jk}$. Which consider the effective mass associated with the in-plane motion of the carriers and the effective mass along the layered axis z. The localization of the superconducting planes along the z axis can be taken into account by changing the z derivative of the order parameter by a finite difference. We obtain

$$F = \int \left\{ \frac{B^2}{8\pi} + \frac{\hbar^2}{4m_{ab}} \left(\nabla - \frac{i2eA}{\hbar c} \right)^2 \Psi + \eta \left| \Psi_{n+1} - \Psi_n \right|^2 + \alpha \left| \Psi \right|^2 + \frac{1}{2} \beta \left| \Psi \right|^4 \right\} dv \tag{2}$$

Where $\Psi(x, y, ns) \equiv \Psi_n$, ∇ now is a two-dimensional vector and $\eta = \frac{\hbar^2}{4m_z d^2}$. In obtaining this expression we have represented the order parameter as $\Psi = |\Psi| \exp(i\alpha)$, where the phase α allows a gauge invariant transformation $\alpha = \alpha_0 + \frac{2eA_z z}{\hbar c}$.

Expression (2) will be used to obtain the partition function of the considered system.

FORMALISM

In high temperature superconductors it is possible to neglect the fluctuations of the magnetic field. If we take into account the fact that the GL parameter κ of such systems is very large, the Lawrence-Doniach functional describing the system consisting of superconducting layers transforms into

$$F = d \sum_n \int \left\{ \alpha \left| \Psi_n \right|^2 + \eta \left| \Psi_{n+1} - \Psi_n \right|^2 + \frac{1}{2} \beta \left| \Psi_n \right|^4 \right\} d\mathbf{r} \tag{3}$$

Where $\tilde{\alpha} = \alpha[1 - H/H_{C2}(T)]$, $H_{C2}(T)$ being the upper critical field. The periodicity of the system along the z direction implies that the order parameter at the (n+1)-th layer is related with the order parameter at the n-th layer in the same way as electron states in a periodic system. This relation can be expressed mathematically through the Bloch´s theorem. Therefore, we have: $\alpha_{eff} = \tilde{\alpha} + 2\eta(1 - cos(qd))$. Where qd represents the phase difference of the order parameter at adjacent layers. In this case the Lawrence-Doniach functional for the layered system is

$$F = d \sum_n \int \left\{ \tilde{\alpha}_{eff} \left| \Psi_n \right|^2 + \frac{\beta}{2} \left| \Psi_n \right|^4 \right\} dr \tag{4}$$

In order to calculate the thermodynamic characteristics, such as the magnetization and the specific heat, we consider the partition function

$$Z = \prod_n \int_{H_0} D\Psi_n \, D\Psi_n^* \exp\left[- \frac{2\pi l^2 Nd}{T} \sum_n \left[\alpha_{eff} \overline{\left| \Psi_n \right|^2} + \frac{\beta}{2} \overline{\left| \Psi_n \right|^4} \right] \right] \tag{5}$$

Where H_0 is the space spanned by the Landau levels of the carriers and $l \equiv \sqrt{c\hbar/2eH}$ is the magnetic length corresponding to a carrier of charge $2e$. Here we suppose the magnetic field so large, that only the lowest Landau levels are occupied (lowest Landau level (LLL) approach). In such approximation the order parameter can be expressed in the symmetric gauge and we obtain the following expression for the partition function:

$$Z \propto \int dU \;\; \exp\left[\frac{NTf(g,U)}{T}\right] \tag{6}$$

With

$$f(g,U) = -\frac{1}{2}g^2U^2 + \frac{1}{2}gU\sqrt{g^2U^2+2} + \sinh^{-1}\left(\frac{gU}{\sqrt{2}}\right) - s(U) \tag{7}$$

And

$$g = \alpha_{eff}\sqrt{2\pi l^2 d/2\beta T} = A\left[1 + \frac{4\eta}{\tilde{\alpha}}\sin^2\left(\frac{qd}{2}\right)\right]t \tag{8}$$

Where $A = \tilde{\alpha}\sqrt{\phi_0 d/2\beta}$ and $t = (T\text{-}T_c(H))/(TH)^{1/2}$. Here ϕ_0 is the flux quantum. We can see that the effective free energy (7) scales with the variable t as a 2D system. Let us note that in the case when the phase difference of the order parameter between adjacent layers is zero ($qd = 0$), our results reduce to that obtained by Tesanovic and Andreev [3].

RESULTS

We evaluate the magnetization ($M = \partial F/\partial H$) and the specific heat ($C = T\partial^2 F/\partial H^2$) of the layered system in the transition region, F being the thermodynamic potential obtained from the partition function (6). We obtain after some algebra

$$\frac{M(H,T)}{\sqrt{TH}}\frac{\phi_0 s H_{c2}'}{A} = gU^2 - U\sqrt{g^2U^2+2} \tag{10}$$

And

$$C(T,H)\frac{s\phi_0}{A^2T} = \left(1 - \frac{gU}{\sqrt{g^2U^2+2}}\right)\left[U^2 + 2\left(gU - \sqrt{g^2U^2+2}\right)\frac{dU}{dg} + Q\right] \tag{11}$$

These expressions generalize those obtained in [3] and take into consideration the variation of the variable U with the scaling variable t. Formulas (10) and (11) are the main results of our analysis.

In Figure 1 we have plotted the dependence of the magnetization M on the scaling variable $2^{1/2}g$ for different values of qd. For temperatures less than the critical temperature the magnetization increases linearly with the scaling variable t. For the case $qd=0$ (center of the Brillouin minizone) this result is in agreement with previous calculations [1-3]. We see that as qd increases, the slope of the magnetization as function of $t=(T\text{-}T_c)/(TH)^{1/2}$ increases. This behaviour can be understood if we take into consideration the fact that at $qd=\pi$ we have the effective magnetization of two superconducting systems with period $2d$ with order parameters oscilating out of phase.

In Figure 2 the dependence of the specific heat C on the scaling variable $2^{1/2}g$ for different values of qd has been plotted. We see that the out of phase values of the order parameter

($qd \neq 0$) results in a wider transition region for the specific heat. Additionally, the specific heat in the close vicinity of the critical temperature diminishes in comparison with the in-phase value ($qd=0$). This is in agreement with the above mentioned fact that as qd tends to the edge of the mini-Brillouin zone, the thermodynamic characteristic into consideration is the result of the combined action of two superconducting systems with out of phase order parameters.

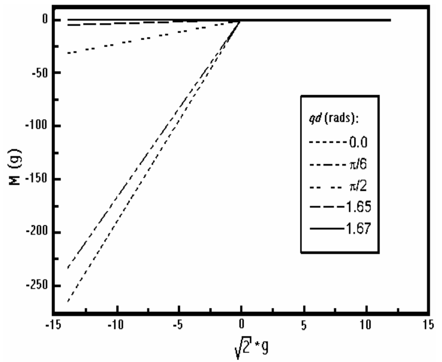

Figure 1: Magnetization M vs scaling variable $2^{1/2}g$ for different values of qd.

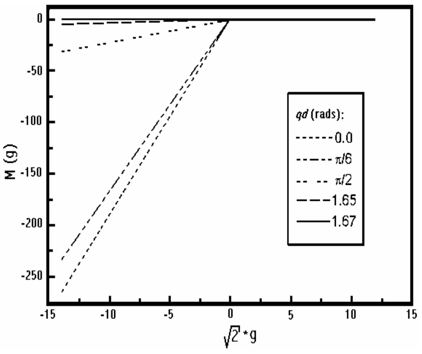

Figure 1: Magnetization M vs scaling variable $2^{1/2}g$ for different values of qd.

CONCLUSIONS

With the aid of the Lawrence-Doniach functional, in the lowest Landau level aproximation, we have obtained explicit expressions for the scaling free energy, magnetization and specific heat of a superconducting anisotropic system in a perpendicular external magnetic field. We have shown that the phase difference of the order parameter between adjacent layers affects the critical behaviour of thermodynamic magnitudes such as the magnetization and the specific heat. In this discussion we have limited ourselves to the limit of strong magnetic fields. The extension to low fields is more complex ad requires the use of methods beyond the LLL approximation [12].

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