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FAR FIELD PROPERTIES OF SPATIAL COHERENCE BEAMS

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ABSTRACT

Spatial coherence of optical fields can be considered as beam structured if the size of the coherence patch varies slowly through the propagation of the optical field. As a consequence, the correlation of the optical field will be concentrated in a finite region around the direction of propagation. For properly describing it in the Fraunhofer-Fresnel domain, the marginal cross-spectral density is introduced. The superposition of spatial coherence beams in this domain is also analysed. It produces an interference field and a spatial coherence Moiré.

DEFINITION OF SPATIAL COHERENCE BEAMS

Optical fields can take the form of beams that come as close as possible to spatially localised and non-diverging waves. As a consequence, the beam power is principally concentrated within a small region surrounding the beam axis [1]. Similarly, the spatial coherence of optical fields can be considered as beam structured if the coherence patches in two planes along the direction of propagation are comparable in size. We call these structures *Spatial Coherence Beams.* They are quite different from the conventional spatially coherent beams, which are beams conformed by spatially coherent optical fields [2]. However, the structure of spatial coherence beams at a specific plane is described by the corresponding cross-spectral density [3].

Specifically, let us assume an optical field, whose cross-spectral density is given by

$$W_0\left(\vec{\xi}_1,\vec{\xi}_2\right)=\mu\left(\vec{\xi}_1,\vec{\xi}_2\right)\sqrt{I\left(\vec{\xi}_1\right)}\sqrt{I\left(\vec{\xi}_2\right)}, \tag{1}$$

illuminates an aperture of complex transmission $t\left(\vec{\xi}\right)=\left|t\left(\vec{\xi}\right)\right|e^{i\phi\left(\vec{\xi}\right)}$ (Fig. 1).. $\mu\left(\vec{\xi}_1,\vec{\xi}_2\right)=\left|\mu\left(\vec{\xi}_1,\vec{\xi}_2\right)\right|e^{i\,\alpha\left(\vec{\xi}_1,\vec{\xi}_2\right)}$ denotes its complex degree of spatial coherence [3], and $I\left(\vec{\xi}\right)$ its intensity distribution across the object transmission. Note that $0\leq\left|\mu\left(\vec{\xi}_1,\vec{\xi}_2\right)\right|\leq 1$ and $\mu\left(\vec{\xi},\vec{\xi}\right)=\left|\mu\left(\vec{\xi},\vec{\xi}\right)\right|=1$. Then, the cross-spectral density of the field that emerges from the aperture will be given by

$$W_{AP}\left(\vec{\xi}_1,\vec{\xi}_2\right)=W_0\left(\vec{\xi}_1,\vec{\xi}_2\right)t\left(\vec{\xi}_1\right)t^*\left(\vec{\xi}_2\right). \tag{2}$$

So, the optical field propagation will be described by a formula, due first to Zernike [4] that relates $W_{AP}\left(\vec{\xi}_1,\vec{\xi}_2\right)$ to the cross-spectral density of the field at the observation plane, $W_{OP}\left(\vec{r}_1,\vec{r}_2\right)$. In paraxial approach (Fraunhofer-Fresnel domain), this formula takes the form [5]

$$W_{OP}\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\right)=\left(\frac{1}{\lambda z}\right)^2 e^{i\frac{k}{z}\vec{r}_A\cdot\vec{r}_D}\int e^{-i\frac{k}{z}\vec{\xi}_A\cdot\vec{r}_D} \int W_{AP}\left(\vec{\xi}_A+\frac{1}{2}\vec{\xi}_D\,,\vec{\xi}_A-\frac{1}{2}\vec{\xi}_D\right)e^{i\frac{k}{z}\left(\vec{\xi}_A-\vec{r}_A\right)\vec{\xi}_D}\,d^2\xi_D\,d^2\xi_A \quad ,(3)$$

- .λ denotes the mean wavelength of the optical field and $k=\dfrac{2\pi}{\lambda}$.
- $\vec{\xi}_A=\frac{1}{2}\left(\vec{\xi}_1+\vec{\xi}_2\right)$ and $\vec{\xi}_D=\vec{\xi}_1-\vec{\xi}_2$, where the position vectors $\vec{\xi}_j$ (j=1,2) denote an arbitrary pair of points inside the aperture.
- $\vec{r}_A=\dfrac{1}{2}\left(\vec{r}_1+\vec{r}_2\right)$ and $\vec{r}_D=\vec{r}_1-\vec{r}_2$ where the position vectors $\vec{r}_j$ (j=1,2) denote an arbitrary pair of points on the observation plane.
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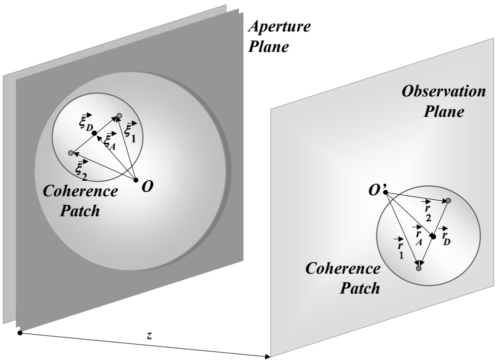

Fig. 1: Centre and difference co-ordinates in both the aperture plane and the observation plane

$$W\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\,;\vec{\xi}_A\right)=e^{-i\frac{k}{z}\vec{r}_D\cdot\vec{\xi}_A}\int W_{AP}\left(\vec{\xi}_A+\frac{1}{2}\vec{\xi}_D\,,\vec{\xi}_A-\frac{1}{2}\vec{\xi}_D\right)e^{i\frac{k}{z}\left(\vec{\xi}_A-\vec{r}_A\right)\vec{\xi}_D}\,d^2\xi_D \quad (4)$$

Specifies the contribution to the correlation of the optical field at locations $\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D$ inside each coherence patch in the observation plane, due to the correlation of the optical field at all the location pairs $\vec{\xi}_A+\frac{1}{2}\vec{\xi}_D\,,\vec{\xi}_A-\frac{1}{2}\vec{\xi}_D$ inside the coherence patch specifically centred on $\vec{\xi}_A$ in the aperture.

If the eq.(3) is expressed in terms of $W\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\,;\vec{\xi}_A\right)$, then

$$W_{OP}\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\right)=\left(\frac{1}{\lambda z}\right)^2 e^{i\frac{k}{z}\vec{r}_A\cdot\vec{r}_D}\int W\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\,;\vec{\xi}_A\right)d^2\xi_A\,\cdot \tag{5}$$

Because of the definition of $W\left(\vec{r}_A+\frac{1}{2}\vec{r}_D\,,\vec{r}_A-\frac{1}{2}\vec{r}_D\,;\vec{\xi}_A\right)$ given by eq.(4), we call it the marginal cross-spectral density. Except for the phase coefficient, it is a Fourier transform of the spectral-cross density of the optical field that emerges from the aperture, which mathematically resembles a Wigner distribution function [6].

Therefore, because of the properties of the Fourier transform, the radius of the coherence patch in the aperture, $\delta\xi$, and the radius of the support of the marginal cross-spectral density, $\delta\, r$, must satisfy a spatial uncertainty relationship.

$$\delta\xi\;\delta\, r\geq\frac{\lambda z}{4\pi}\cdot \tag{6}$$

The set of comparable values of $\delta\xi$ and $\delta\, r$ can be chosen by comparing the support of the complex degree of spatial coherence at both the aperture and the observation planes

INTERFERENCE FIELDS AND MOIRÉ PATTERNS

Let us replace the aperture for a pair of pinholes with separation vector $\vec{a}$, centred at the location $\vec{\xi}'_A$ on the aperture plane. Its transmission will be given by $t(\vec{\xi}_A)=\delta(\vec{\xi}_A-\vec{\xi}'_A)\left[\delta\left(\vec{\xi}_A-\frac{\vec{a}}{2}\right)+\delta\left(\vec{\xi}_A+\frac{\vec{a}}{2}\right)\right]$, with $\delta(\circ)$ the Dirac's delta function [8]. Then, the intensity distribution at the observation plane will be given by

$$\begin{aligned}I_{OP}(\vec{r}_A)=W_{OP}(\vec{r}_A\,,\vec{r}_A)=\left(\frac{1}{\lambda\, z}\right)^2&\left[I\left(\vec{\xi}'_A+\frac{\vec{a}}{2}\right)+I\left(\vec{\xi}'_A-\frac{\vec{a}}{2}\right)\right]\\&\left\{1+2\,\frac{\sqrt{I\left(\vec{\xi}'_A+\frac{\vec{a}}{2}\right)}\sqrt{I\left(\vec{\xi}'_A-\frac{\vec{a}}{2}\right)}}{I\left(\vec{\xi}'_A+\frac{\vec{a}}{2}\right)+I\left(\vec{\xi}'_A-\frac{\vec{a}}{2}\right)}\left|\mu(\vec{a})\right|\cos\left[\frac{k}{z}\left(\vec{\xi}'_A-\vec{r}_A\right)\vec{a}+\alpha(\vec{a})\right]\right\}\end{aligned} \tag{7}$$

So, it is apparent that each pair of points inside a specific aperture coherence patch provides a Young interference pattern at the observation plane, whose visibility and support depend on the intensity distribution across the patch and the modulus of the complex degree of spatial coherence. The separation vector $\vec{a}$ will determine the period and orientation of the interference fringes. The specific location of the patch centre $\vec{\xi}'_A$ and the phase $\alpha(\vec{a})$ will introduce fringe shifts. If both the points are not included inside the coherence patch, then the value of the modulus of the complex degree of spatial coherence will be negligible. As a consequence, they cannot generate *a Young interference pattern* on the observation plane.

On the other hand, let us consider at the first the contributions from two pinhole pairs, located inside two disjoint aperture coherence patches, say centred at $\vec{\xi}_A^{(1)}$ and $\vec{\xi}_A^{(2)}$ respectively, on the marginal cross-spectral density at the observation plane. The vector separations will be $\vec{a}_1$ and $\vec{a}_2$ respectively, so that their transmissions will be given by $t_j\left(\vec{\xi}_A\right)=\delta\left(\vec{\xi}_A-\vec{\xi}_A^{(j)}\right)\left[\delta\left(\vec{\xi}_A-\frac{\vec{a}_j}{2}\right)+\delta\left(\vec{\xi}_A+\frac{\vec{a}_j}{2}\right)\right]$, with j=1,2.

Furthermore, let us denote $I_T^{(j)}\left(\vec{\xi}_A\right)=I\left(\vec{\xi}_A+\frac{\vec{a}_j}{2}\right)+I\left(\vec{\xi}_A-\frac{\vec{a}_j}{2}\right)$. Therefore,evaluating it for $\vec{r}_D=0$ we obtain the final intensity distribution at the observation plane, that is

$$I_{OP}\left(\vec{r}_A\right)=W_{OP}\left(\vec{r}_A,\vec{r}_A\right)=\left(\frac{1}{\lambda z}\right)^2\left[I_T^{(1)}\left(\vec{\xi}_A^{(1)}\right)+I_T^{(2)}\left(\vec{\xi}_A^{(2)}\right)\right]$$

$$\left\{1+2\frac{\sqrt{I\left(\vec{\xi}_A^{(1)}+\frac{\vec{a}_1}{2}\right)}\sqrt{I\left(\vec{\xi}_A^{(1)}-\frac{\vec{a}_1}{2}\right)}}{I_T^{(1)}\left(\vec{\xi}_A^{(1)}\right)+I_T^{(2)}\left(\vec{\xi}_A^{(2)}\right)}\left|\mu\left(\vec{a}_1\right)\right|\cos\left[\frac{k}{z}\left(\vec{\xi}_A^{(1)}-\vec{r}_A\right)\vec{a}_1+\alpha\left(\vec{a}_1\right)\right]\right.$$

$$\left.+2\frac{\sqrt{I\left(\vec{\xi}_A^{(2)}+\frac{\vec{a}_2}{2}\right)}\sqrt{I\left(\vec{\xi}_A^{(2)}-\frac{\vec{a}_2}{2}\right)}}{I_T^{(1)}\left(\vec{\xi}_A^{(1)}\right)+I_T^{(2)}\left(\vec{\xi}_A^{(2)}\right)}\left|\mu\left(\vec{a}_2\right)\right|\cos\left[\frac{k}{z}\left(\vec{\xi}_A^{(2)}-\vec{r}_A\right)\vec{a}_2+\alpha\left(\vec{a}_2\right)\right]\right\}.\quad(8)$$

These results show that the superposition of marginal cross-spectral densities from different coherent patches produces a *(Spatial Coherence) Moiré pattern* on the observation plane [7]. This work was partially performed at the Abdus Salam International Centre for Theoretical Physics (AS-ICTP, Trieste – Italy). One of the authors, Francisco F. Medina-Estrada, undertook this work with the support of the “ICTP Programme for Training and Research in Italian Laboratories”, Trieste, Italy..

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