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EFFECTS OF GEOMETRICAL PARAMETERS ON CARRIER STATES IN A 2D-QUANTUM RING

R. D. Guerrero, J. C. Granada, N. Porras-Montenegro
Departamento de Física de la Universidad del Valle

ABSTRACT

In this work we examine the carrier states in a 2D-Quantum Ring, using the effective mass-approximation. In this system carriers are confined in the radial direction by a finite square potential well. We find the exact solution of the time-independent Schrödinger equation for the system, we determine the energy spectrum and we find the density of probability for a carrier confined in such ring, Also, we study the effects of the geometrical parameters on the energy spectrum of a carrier and examine how the energy spectrum is re-structured by a static magnetic field applied in the normal direction to the system.

RESUMEN

En este trabajo se estudian los estados de portadores en un anillo cuántico bidimensional, con base en la aproximación de la masa efectiva. En este sistema los portadores están confinados en la dirección radial por un pozo finito de potencial. Se halla la solución exacta de la ecuación de Schrödinger independiente del tiempo, se determina el espectro de energía y se encuentra la densidad de probabilidad para un portador confinado en tal sistema. Adicionalmente se consideran los efectos de los parámetros geométricos en el espectro de energía de un portador y se examina como el espectro es re-estructurado por un campo magnético estático aplicado en la dirección normal al sistema.

Key words. Quantum ring, carrier states, effective mass approximation

INTRODUCTION

Semiconductors nanorings form quantum dot structures in which quantum confinement produces levels of discrete energy within the bulk band gap, which have been seen low-temperature photoluminescence studies. Recently, self-assembled semiconductor nanorings have been growth in InAs/GaAs by molecular-Beam epitaxy. However, well-controlled methods for fabricating such structures remain a challenge [1]. Early experiments showed that the electron beam could be used for direct-write inorganic lithography with high resolution, and a variety of materials have been used. Demonstrations of pattern ring formation have been made [1]. Using a scanning transmission electron microscope (STEM). in these methods, electron transparent specimens were prepared by picking up small pieces of crushed glass suspended in acetone using a carbon film with holes supported by a copper grid. This method of preparation cannot provide large thin areas of uniform thickness, although variation in thickness has little effect on the diameter of individual quantum ring. This geometry had not been achieved in semiconductors for sizes such that the electrons would propagate coherently (non-diffusively) through the ring [1].

In this work, in order to contribute to the understanding of the carrier state in QRs, we calculate the effects of the geometrical parameters and the quantum flux through the QR on the energy of the carrier states in this system. In our calculations we use the effective mass-approximation, considering that the size of the QR is larger than the size of the unitary cell.

THEORETICAL MODEL. Zero magnetic field.

In the first approach we will describe a free particle moving in a circularly symmetric region $r_0<r<r_0+\Delta r$, where ρ_0 is the inner radius of the quantum ring and Δr is its width. For a charge carrier in the QR (zero magnetic field) the Schrödinger equation in polar coordinates (r,q) and in atomics units is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\ \frac{\partial y}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 y}{\partial q^2}+2Ey\ (r,q)-V(r)y\ (r,q)=0$$

Where:

$$V(r)=\begin{cases}0, & r_0\le r\le (r_0+\Delta r)\\ \infty, & Otherwise\end{cases}$$

Here the distances are measured in effective Bohr radius $a_B=\varepsilon\hbar^2/(me^2)$ and the energy is measured in effective Rydbergs $Ry=e^2/(2\varepsilon a_B)$, we will to consider the electron effective mass equal to $0.065{*}m_0$ and the relative dielectric constant for GaAs is equal to 12.58. In this system the angular momentum is conserved, then we will use the wave functions of the form. $y\ (r,q)=F(q)R(r)$. Replacing this wave function in the Schrödinger equation, we obtain the following equations

$$\frac{\partial^2 F}{\partial f^2}+l^2F(f)=0 \qquad and \qquad \frac{\partial}{\partial r}\left(r\ \frac{\partial R}{\partial r}\right)+\left(k^2-\frac{l^2}{r}\right)R(r)=0.$$

The solution of the angular component equation has the form $F(\phi)=e^{\pm il\phi}/\sqrt{2\pi}$ and it reflects a azimuthal symmetry of the system, that implies the angular momentum conservation. It should by noted that both positive and negative values of the angular momentum are allowed. The solution for the radial equation is given in terms of a linear combination of Bessel function of first and second kind of order equal to l [2]. For the simple case of infinite barrier potential well, the boundary condition imposed on the wave function determine that the radial component of the wave function is given by

$$R_n(r)=B_n\left\{Y_n(k_r r)-\frac{Y_n(k_r r_0)}{J_n(k_r r_0)}J_n(k_r r)\right\}$$

where B_n is a normalization constant for the n state to the system. For symmetry reasons we will write the kinetic energy of a free particle in the ring, without magnetic field by $E_k=\{k_{rn}^2+l^2/(2r_0^2)\}$ where $k_{\rho n}$ is the value of k_ρ that is satisfied by the radial eigenvalue equation for the state n of the system. For the finite wall potential well the radial confinement potential is given by $V(\rho)=V_0\theta(\rho_0-\rho)\,\theta(\rho-\rho_0-\Delta\rho)$ where the V_o is calculated for a 30% Al concentration how in reference [3]. The wave function is given by.

$$R(r)=\begin{cases}C_{1n}I_n(k_1 r), & 0\le r<r_0\\ Y_n(k_r r)-\dfrac{Y_n(k_r r_0)}{J_n(k_r r_0)}J_n(k_r r), & r_0\le r\le r_0+\Delta r\\ C_{1n}K_n(k_1 r), & r>r_0+\Delta r\end{cases}$$

Where $k_1^2=2(V_0-E)\quad k_1^2=2E$. The matching condition at the boundaries of the ring give the transcendental equation for the system, in the finite wall potential well.

Non-zero magnetic field. In this case the Hamiltonian for the system in atomic units is,

$$H=\left(-i\nabla+\tfrac{f}{r}\right)^2+V(r)$$

Where ϕ is the number of quantum flux across the surface of the ring, For infinite wall potential well, in the region $\rho_0<\rho<\rho_0+\Delta\rho$. where angular momentum conservation is accounted. We are looking for solutions in the form $y\,(r,q)=R(r)\frac{e^{ilq}}{\sqrt{2p}}$ we take in to account the angular momentum conservation. Then function *R(r)* satisfies the equation [4].

$$\frac{\partial^2 R}{\partial r^2}+\frac{1}{r}\frac{\partial R}{\partial r}+\left[e-\left(\frac{m+f}{r}\right)^2\right]R(r)=0\,,$$

the solution is given in terms of a linear combination of Bessel functions of first and second kind, like

$$R_n(r,f)=B_n\left\{Y_{-(n+f)}(k_r\,r)-\frac{Y_{-(n+f)}(k_r\,r_0)}{J_{(n+f)}(k_r\,r_0)}J_{(n+f)}(k_r\,r)\right\}.$$

Applying the infinite wall potential well boundary conditions, we obtain the transcendental equation to obtain the energy eigen-values for the carrier in the QR.

RESULTS AND DISCUSSION

In the Fig. (1) we represent the influence of the of the geometrical parameters on the energy eigen-values of the ground and few excited states of an electron in GaAs(Ga,Al) As quantum ring at zero magnetic field. As it is observed in Fig. 1(a), the energy of the electron in the QR increases with the order of the excited state. For large values of the radius of the ring this energy reaches a constant value. In Fig. 1(b) the transition energies diminishes with the radius of the QR, also reaching a constant value for large radius of the QR. In Fig. 1(c) we show how the energy of the electron for a constant for a constant value of the radius of the QR diminishes with the width of the QR; this implies that the transition energies go to zero for large values of the width of the QR, Fig. 1(d).

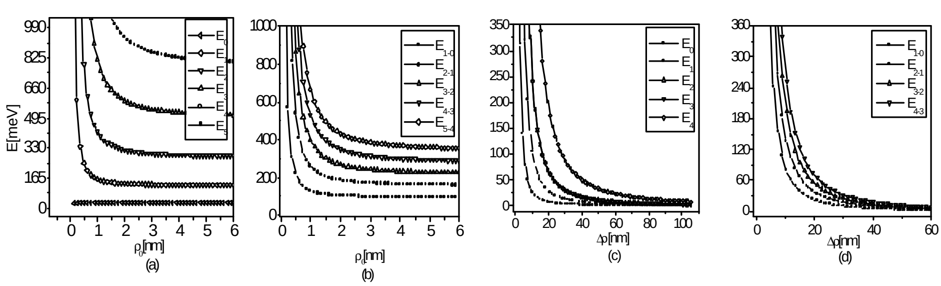
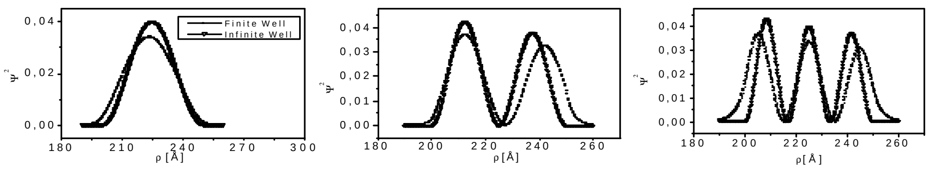


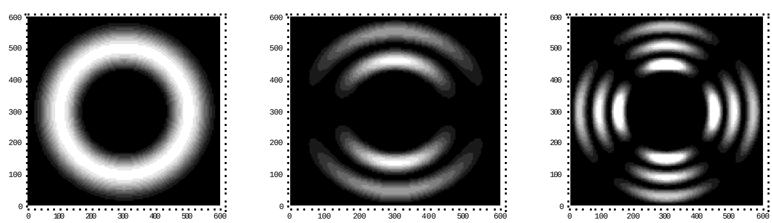
Figure 1.(a),. Effects of the Ring width on the electron energy eigen-values in a GaAs(Ga,Al) as quantum ring (b).Transition energies between different electron states, for a fixed $\Delta\rho$=3nm. (c). Effects of the Ring radius on the electron energy eigen values in a GaAs(Ga,Al) as quantum ring, (d). Transitions energy between electron states., for a fixed ρ_0=20nm

In Fig. 2 we compare the radial probability distribution of a carrier in the QR for finite and infinite potential wall well confinement. As observed this probability distribution is higher in the infinite potential well for every carrier state. In Fig. 3 we represent the polar probability distribution for a carrier in the QR. In Fig. 4 we represent the effects of the geometrical parameters and of the quantum flux through the QR on the energy eigen-value of the ground state of an electron in this structure. As it is observed in Fig. 4(a) this energy diminishes with the width of the QR and is higher for large values of the quantum flux. For constant value of the width of the QR, in Fig. 4(b), we observe the energy of the ground state diminishes with the radius of the QR, being higher for larger values of the quantum flux, however for zero quantum flux, this energy increases with the radius. In all cases, in the limit of large radius and large width of the QR, the energy of the electron gets the values of the first Landau Level.



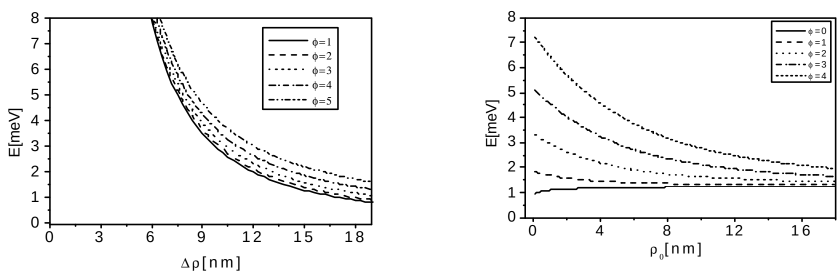
(a) (b) (c)

Figure 2. Radial probability distribution for a carrier in the quantum ring with a characteristic radius of 20[nm] and a ring width of 6[nm]. (a) Ground state, (b) First exited state, (c) Second excited state.



(a) *(b)* *(c)*

Figure 3. Polar distribution of probabilities for the carrier in the ring. (a) Ground state. (b) First excited state. (c) Second excited



(a) (b)

Figure 4.(a). Effects of the magnetic flux and the ring width on the energy eigenvalue for an electron on the ground state in the QR, with ρ_0=20nm (b) Effects of the quantum flux on the energy eigenvalue of an electron in the ground state in a ring with $\Delta\rho$=15nm and ρ_0=20nm.

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