REVISTA COLOMBIANA DE FÍSICA, VOL. 37, No. 1. 2005

BEAM PROPAGATION AND SURFACE WAVE FORMATION ALONG THE INTERFACE OF PHOTOREFRACTIVE MEDIA

Rafael Torres-Cordoba
Universidad Autonoma de Ciudad Juarez, Av. Del Charro 450 norte, C.P. 32310,
Cd. Juarez Chih. Mexico.

RESUMEN

Se considera la formación de ondas superficiales a través de la interfase entre dos medios, donde dichas ondas pueden ser caracterizadas por la orientación del cristal, el coeficiente no lineal de difusión y la amplitud del haz incidente. Estas ondas espaciales pueden ser guiadas a lo largo de las interfaces siguientes: PRC`-PRC (PRC representa el cristal foto-refractivo), PRC – metal ideal y PRC - dieléctrico óptico lineal y es en donde se producen estas ondas foto-refractivas superficiales autoinducidas, resultando de un análisis electromagnético Eigen modal. Este confinamiento de la luz se predice con evidencias teóricas.

ABSTRACT

We have considered the surface wave formation along the interface of two different media, characterized by the crystal orientation, the strength of diffusion photorefractive non-linearity (of the gradient type), and the incident beam amplitude. These spatial waves can be guided along: PRC`-PRC (PRC is a photorefractive crystal), PRC – ideal metal and PRC - optically linear dielectric media boundaries; these are used to produce self-induced photorefractive surface waves, resulting from an electromagnetic Eigen-mode analysis, thus the light self-confinement phenomenon takes place. Theoretical evidence is given.

I. - INTRODUCTION

The propagation of spatial solitons in photorefractive crystals has generated much interest in recent years, as a way to obtain the self-confinement of the incident light beam in the propagation media. Nowadays, the spatial soliton propagation – due to quasi-local drift mechanisms of photorefractive non-linearity – has been obtained both experimentally and theoretically [1, 2, 3, 4]. The spatial soliton formation results from the balance between linear diffraction and non-linear self-focusing, and several applications have been developed.

Alternate approaches to the generation of soliton-like beams may be based on non-linear surface waves. One way is when a beam falls on the interface of two different media; that is, on the wave trapped on the interface between either a crystal and a metal, or a crystal with opposite sign of the non-linear diffusion, a dielectric with a lower average refractive index and a crystal. The occurrence of the self-confinement and the stability of the incident beam on the interface are dependent on the non-linearity diffusion (of the gradient type) of the photorefractive crystals [5].

Theoretical predictions of these waves were obtained as follows: The guided, spatially confined surface wave along the boundary between IM-PRC, OLD-PRC, (IM is an ideal metal and OLD optically linear dielectric), and PRC`-PRC (with the opposite sign of the non-linearity diffusion) has been previously predicted [5]. In this paper it was assumed that the boundary between two different media does not posses any particular properties, i.e., surface trapping centers, blocking properties for the charge transfer, or surface photo-galvanic effects, etc. Surface wave formation is an Eigen-mode problem, and the dark conductivity influence was considered in [6].

Experimentally, these waves were obtained as a result of a balance of the laser beam self-bending by total internal reflection at the crystal surface to produce self-induced photorefractive surface waves [7].

Surface wave propagation in $Bi_{12}TiO_{20}$ and $Bi_{12}SiO_{20}$ crystals occurs when such waves are induced with a smaller diameter Gaussian beam and energy losses (due to leakage into the crystal volume) are present [8, 9].

Up until now, there is no completely developed theory that allows the simulation of surface wave formation through a mathematical model, where it can be used as a form of soliton-like spatial propagation approximation.

The primary goal of our paper is to develop an analytical approach to the practically important problem of considering beam propagation along two media interfaces. We finally ponder if it is possible to confine light within the small area along photorefractive media surface.

For symmetrical and anti-symmetrical surface wave formations, the beam propagates along the interface of two PRC`-PRC media with the opposite sign of the non-linearity diffusion. Then the surface wave is formed by diffraction and the guided mode under consideration is formed as an interference pattern of two waves deflected in opposite directions. For IM-PRC media, surface waves are produced due to the coupling action of reflection, non-linearity diffusion, and diffraction.

Non-linearity diffusion is described by spatially varying dielectric constant $\varepsilon(x,z)=\varepsilon+\delta\varepsilon(x,z)$ [5], where $\delta\varepsilon(x,z)$ represents photo-induced changes given the non-linearity diffusion. This nonlinearity produces the beam self-bending.

Below, we describe a mathematical model to obtain the differential equation, which describes the beam amplitude behavior in each medium, the continuity conditions that satisfy the Maxwell equations in the interface media, the conditions that are necessary for the surface wave formation, and finally the conclusions.

II. - MATHEMATICAL MODEL

In figures 1 and 2 the beam propagation along the interface of the media is shown. Another simplification is that surface current, flowing exactly on the boundary surface [10], can also be neglected in the paraxial approximation.

For PRC`-PRC media, in the $x<0$ region it is the first PRC` and in the $x\geq 0$ region it is the second PRC. Both are in contact with the defined orientation crystal.

The wave equation below is valid for the $x\geq 0$ region. It is a standard scalar three-dimensional equation for the complex amplitude $E_1(x,y,z,t)$, propagating in an unbounded, optically transparent medium with some spatial variations of the dielectric constant $\varepsilon(x,z)$ [5].

$$\nabla^2 E_1(x,y,z,t) - \mu\varepsilon_0\varepsilon(x,z)\frac{\partial^2 E_1(x,y,z,t)}{\partial t^2} = 0 \qquad (1)$$

Further, assume a 2D light beam propagation of the monochromatic wave with frequency ω, and $\varepsilon(x,z) = \varepsilon + \delta\varepsilon(x,z)$, where the positive sign is for the $x \geq 0$ region and the negative sign is for the $x<0$ region, ε is the spatially uniform average dielectric constant and $\delta\varepsilon(x,z)$ is the photo-induced change, $\varepsilon >> |\delta\varepsilon(x,z)|$ [5].

Linear light polarization is considered and the electric field is in the axis x-z plane (see Figure 1), then assuming electric field

$$E_1(x,y,z,t)=E(x,z)\exp(-i\beta z-i\omega t) \qquad (2)$$

where β is the wave propagation constant, then substituting in (2) into (1), for $x \geq 0$ we obtain**:**

$$\frac{\partial^2 E(x,z)}{\partial x^2} + \frac{\partial^2 E(x,z)}{\partial z^2} - 2i\beta\frac{\partial E(x,z)}{\partial z} - (\beta^2 - \omega^2\mu\varepsilon\varepsilon_0)E(x,z) = 0 \qquad (3)$$

For calculation of $\delta\varepsilon(x,z)$ we use Kukhtarev's equations [11] for a purely electronic type of photoconductivity.

$$\frac{\partial N_D^i}{\partial t} = s_e I(N_D - N_D^i) - \gamma_e n_e N_D^i \qquad (4)$$

$$j_e = e n_e \mu_e E_{SC} + \mu_e k_B T \nabla n_e \qquad (5)$$

$$\frac{\partial n_e}{\partial t} = e^{-1}\nabla j_e + s_e I(N_D - N_D^i) - \gamma_e n_e N_D^i \qquad (6)$$

$$\nabla E_{SC} = -\varepsilon_0\varepsilon^{-1} e(n_e + N_A - N_A^i) \qquad (7)$$

where N_D is the number of dopants per unit volume, N^i_D is the number of ionized dopants per unit volume, s_e is the excitation cross section, γ_e is the recombination coefficients, n_e is the density of free electrons, I is the light intensity, μ_e is the electron mobility, j_e is the current density, and E_{SC} is the electric field of the space charge.

With the steady-state conditions of illumination (when nothing changes with time), from (4) and (6), we obtain; $\frac{\partial N_D^i}{\partial t} = \frac{\partial n_e}{\partial t} = 0$, then $\nabla j_e=0$, but $j_e=0$, as a result of drift-diffusion equilibrium, the space charge electric field takes the form:

$$E_{SC} = -k_B T e^{-1} n_e^{-1} \nabla n_e \qquad (8)$$

For 2D of the photorefractive charge transport model, $\nabla n_e \to \frac{\partial n_e(x,z)}{\partial x}$ and, in Poisson equation $\nabla E_{SC} \to \frac{\partial E_{SC}(x,z)}{\partial x} = -\varepsilon^{-1} e n_e$, ($N_A=N^i_D$). The density of compensative acceptors N_A equals the density of ionized photorefractive dopants in the dark [11], and for one sided abrupt junction $N_A >> N_D$ and (8) transforms:

$$E_{SC}(x,z) = -k_B T e^{-1} (I(x,z))^{-1} \frac{\partial I(x,z)}{\partial x} \tag{9}$$

Taking into account that for a planar wave

$$(I(x,z))^{-1} \frac{\partial I(x,z)}{\partial x} = 2(E(x,z))^{-1} \frac{\partial E(x,z)}{\partial x}$$

and that transformation of the space charge electric field relief to the index changes is ensured via the linear electro-optic effect [5], the photo-induced change of dielectric constant is $\delta\varepsilon(x,z)=n^4 r E_{SC}(x,z)$; finally, we obtain that:

$$\delta\varepsilon(x,z) = 2n^4 r \frac{K_B T}{e} (E(x,z))^{-1} \frac{\partial E(x,z)}{\partial x} \tag{10}$$

where, r is the electro-optic coefficient determined according to the crystal orientation, K_B is the Boltzman constant, T is the absolute temperature and e the electron charge and n is the average refractive index of the sample ($n^2=\varepsilon$). Parameter $\gamma = k_0 n^2 r K_B T e^{-1}$ characterizes diffusion intensity, $k_0=\omega\sqrt{\mu\varepsilon\varepsilon_0}$ is the wave number in an optically linear medium.

Now, substituting the last equation (10) into (3) and taking into account slow variation of the amplitude

$$\left| \frac{\partial^2 E(x,z)}{\partial z^2} \right| \ll \left| \beta \frac{\partial E(x,z)}{\partial z} \right| \tag{11}$$

we have;

$$a \frac{\partial^2 E(x,z)}{\partial x^2} + sign(x) D \frac{\partial E(x,z)}{\partial x} - ib \frac{\partial E(x,z)}{\partial z} - CE(x,z) = 0 \tag{12}$$

This differential equation is already normalized and describes the case of PRC`-PRC interface, with coefficient

$$sign(x) = \begin{array}{c} 1 \\ -1 \end{array} \; for \; \begin{array}{c} x \geq 0 \\ x < 0 \end{array} \tag{13}$$

The other parameters are $a=k_0^{-2}$, $b=2\beta\, k_0^{-1}$ and $D=2\gamma\, k_0^{-1}$.

For the case of the interface with Optically Linear Dielectric (OLD), the differential equation for the field in the x<0 region is:

$$a \frac{\partial^2 \mathrm{E(x,z)}}{\partial x^2} - ib \frac{\partial E(x,z)}{\partial z} - CE(x,z) = 0 \tag{14}$$

For the case of the interface with ideal metal in the x<0 regions we have:

$$E_{IM}(x,z)=0 \tag{15}$$

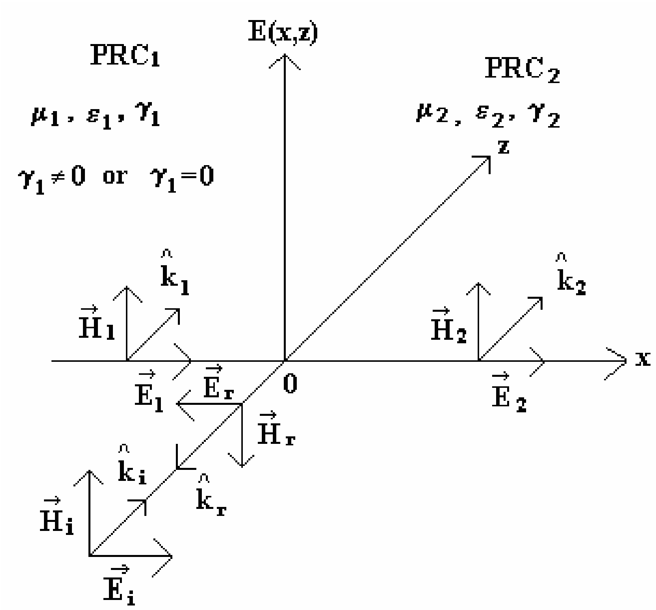

Figure 1. Field continuity where the Maxwell equations are applied in z=0, for PRC`-PRC and OLD-PRC.

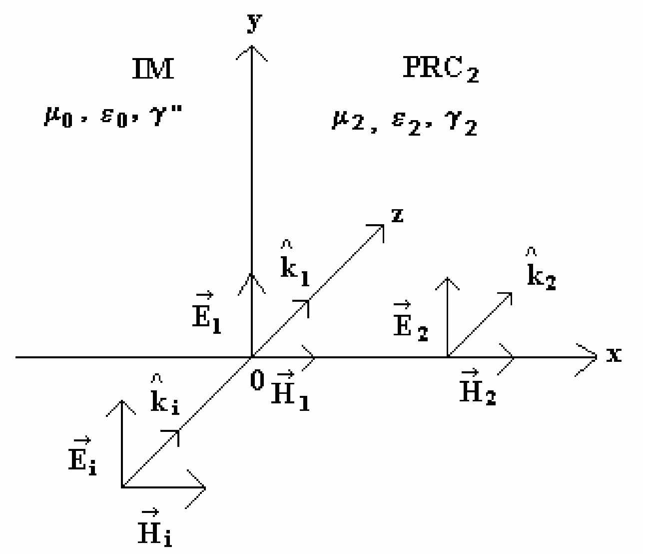

Figure 2. Field continuity where the Maxwell equations are applied in z=0, for IM-PRC (γ_1 represents the infinite conductivity).

III. - CONTINUITY EQUATIONS

If for z>0, E(x,z) has only normal components, i.e. E(x,z) is perpendicular to the (y,z) plane, the following continuity equations must be satisfied. For the PRC`-PRC case:

$$\varepsilon_1(x,z)E_1(x,z)\big|_{x=0} = \varepsilon_2(x,z)E_2(x,z)\big|_{x=0} \tag{16}$$

And

$$\left.\frac{\partial\varepsilon_1(x,z)E_1(x,z)}{\partial x}\right|_{x=0} = \left.\frac{\partial\varepsilon_2(x,z)E_2(x,z)}{\partial x}\right|_{x=0} \tag{17}$$

representing the magnetic field continuity. For OLD-PRC and for z>0 are:

$$\varepsilon_1 E_1(x,z)\big|_{x=0} = \varepsilon_2(x,z)E_2(x,z)\big|_{x=0} \tag{18}$$

And

$$\varepsilon_1\left.\frac{\partial E_1(x,z)}{\partial x}\right|_{x=0} = \left.\frac{\partial\varepsilon_2(x,z)E_2(x,z)}{\partial x}\right|_{x=0} \tag{19}$$

Finally, for IM-PRC we have:

$$\varepsilon_0 E_1(x,z)\big|_{x=0} = \varepsilon_2(x,z)E_2(x,z)\big|_{x=0} = 0 \tag{20}$$

And

$$\left.\frac{\varepsilon_0\partial E_1(x,z)}{\partial x}\right|_{x=0} = \left.\frac{\partial\varepsilon_2(x,z)E_2(x,z)}{\partial x}\right|_{x=0} \tag{21}$$

IV. - SURFACE WAVE SOLUTION

We define the following boundary condition: $G(z)=E(x,z)|_{x=0}$ and $F(z)=\left.\frac{\partial E(x,z)}{\partial x}\right|_{x=0}$ Then solving and substituting these conditions to (12) and also applying the Laplace Transform, $\mathcal{L}$(E(x,z))=E(x,s), for x≥0 we obtain:

$$E(x,z)=\exp(-\frac{xD_2}{2a_2})\left\{G_2(s)\cos\left[B_1(s)x\right]+\left[\frac{D_2}{a_2}G_2(s)+2F_2(s)\right]\frac{\sin\left[B_1(s)x\right]}{B_1(s)}\right\}+g_2(s) \tag{22}$$

where $a=k_{20}^{-2}$, $D_2=2\gamma k_{20}^{-2}$ and

$$B_1(s)=-i\sqrt{\beta^2-k_{20}^2(1-\gamma^2)+2i\beta s} \tag{23}$$

To obtain the conditions of the surface wave formation, it is instructive to consider calculating the limit of the solution z→∞, and then to apply the inverse Laplace Transform E(x,z) = $\mathcal{L}^{-1}$(E(x,s)). Using the final value theorem we have:

$$\lim_{z\to\infty}E_2(x,z)=\lim_{s\to 0}sE_2(x,s)=E(x) \tag{24}$$

where E(x) represents the surface wave. In order to show that the formation of this wave depends on the shape of the incident beam, we return to (22) that depends on the incident beam shape in the function $g_2(x,s)$. Using the Laplace inverse Transform, one can obtain:

$$g_2(x,s) = \sqrt{\frac{b_2}{4\pi z}} \exp(-\eta z) \int_0^x K_1(t) \exp\left\{ -\frac{b_2(x-t)^2}{4z} - \frac{D_2(x-t)}{2} \right\} dt \tag{25}$$

Or

$$g_2(x,s) = \sqrt{\frac{b_2}{4\pi z}} \exp(-\eta z) \int_0^x K_1(t) \mathbf{G}(x,z,t,0) dt \tag{26}$$

Where

$$\mathbf{G}(x,z,t,0) = \sqrt{b(4i\pi z)^{-1}} \exp\left[-\frac{1}{2} ib(t-x)^2 (2z)^{-1} \right] \exp\left[\frac{1}{2} D(t-x) \right] \tag{27}$$

Here $K_1(x)$ is the input optical field distribution and $\eta=(C_2+4^{-1}a_2^{-1}D_2^2)(ib_2)^{-1}$ (represents the wave number that produces the phase shift). For the square beam case, this term does not tend toward zero. Here we could visualize several concepts of interpretation. Eq. (25) contains a propagator of the Kernel function (27) already discussed in [14], now the behavior of $K_1(x)$ could be analyzed as well as in [10], indicating how the beam is spread until it falls slowly or quickly to zero, or it is stabilized, according to the values of the parameters and the beam shape. One could also calculate the limit when g_2 is a function of s:

$$\lim_{z\to\infty} g_2(x,z) = \lim_{s\to 0} s g_2(x,s) = \begin{matrix} g_2(x) \\ zero \end{matrix} \tag{28}$$

Then applying (16) and (17) (when x=0) and simultaneously using conditions of continuity, we have:

$$F_2(s) = -\frac{\alpha_0\alpha_4^{-1}(s+\alpha_1\alpha_0^{-1})F_1(s) + \alpha_2\alpha_4^{-1}(s+\alpha_3\alpha_2^{-1})G_1(s) - \alpha_0\alpha_4^{-1}F_1(z=0) - \alpha_2\alpha_4^{-1}G_1(z=0)}{s+\alpha_5\alpha_4^{-1}} \tag{29}$$

$$G_2(s) = -\frac{\alpha_{02}F_2(s) - \varepsilon_1 G_1(s) - \alpha_{01}F_1(s)}{\varepsilon_2} \tag{30}$$

Coefficients are given in Appendix A.

V. - STEADY STATE SOLUTIONS

Now, calculating the limit in continuity functions, $G_2(z)$ and $F_2(z)$, that fulfill (29) and (30), we apply the Laplace transform and the final value theorem as follows:

$$\lim_{z\to\infty} G_2(z) = \lim_{s\to 0} sG_2(s) = const. \tag{31}$$

$$\lim_{z\to\infty} F_2(z) = \lim_{s\to 0} sF_2(s) = const. \tag{32}$$

and for a specific case when $g_2(x,z\to\infty)=0$, the homogenous solution is:

$$E_H(x,s) = \exp(-\frac{xD_2}{2a_2})\left\{G_2(s)\cos\left[B_1(s)x\right] + \left[\frac{D_2}{a_2}G_2(s) + 2F_2(s)\right]\frac{\sin\left[B_1(s)x\right]}{B_1(s)}\right\} \tag{33}$$

For the formation of the surface waves it is enough that for (31) and (32) at least one be different to zero. The variable coefficient controlling the decay is `$s+\alpha_5\alpha_4^{-1}$` for $sG_2(s)$ and $sF_2(s)$ when $s\to 0$, one or both are different to zero. Then, returning to (12) and applying (22), surface wave solutions are obtained from (23):

$$B_1(s\to 0) = -i\sqrt{\beta^2 - k_{20}^2(1-\gamma^2)} \tag{34}$$

for $B_1\neq 0$,

$$E(x) = \exp(-\frac{D_2x}{2a_2})\left\{A_1\cos(B_1x) + A_2\frac{\sin(B_1x)}{B_1}\right\} \tag{35}$$

and for $B_1=0$,

$$E(x) = \exp(-\frac{D_2x}{2a_2})\left\{A_1 + A_2x\right\} \tag{36}$$

A. Eigen-mode solution for PRC`-PRC

If the coefficient α_5 that represents wave beam decay along the z- axis equals to zero, then we obtain that $\alpha_{02}{}^2C_2+\alpha_{02}D_2\varepsilon_2-\varepsilon_2{}^2a_2=0$. Taking into account that $C_2=(\beta^2-k_{20}{}^2)k_{20}{}^{-2}$ if $k_0{}^{-1}\gamma=n^2rK_BTe^{-1}$, then $\alpha_{02}=2n_2{}^4r_2K_BTe^{-1}= 2n_2{}^2\gamma\ k_{20}{}^{-1}$ ($\alpha_{01}=2n_1{}^4r_2K_BTe^{-1}= 2n_1{}^2\gamma`\ k_{10}{}^{-1}$), but $n_2{}^2=\varepsilon_2$, we obtain:

$$\frac{4\varepsilon_2^2\gamma^2C_2}{k_{02}^2} + \frac{2\gamma\varepsilon_2^2D_2}{k_{20}} - \frac{\varepsilon_2^2}{k_{20}^2} = 4\gamma^2C_2 + 2\gamma\varepsilon D_2k_{20}^2 - 1 = 0 \tag{37}$$

Now, substituting $D_2=2\gamma\ k_{10}{}^{-1}$ and C_2 to (37), we obtain:

$$4\gamma^2\frac{\beta^2-k_{20}^2}{k_{20}^2} + 4\gamma^2 - 1 = 0 \tag{38}$$

For propagation constant we have:

$$\beta^2 = \frac{k_{20}^2}{4\gamma^2} \Rightarrow \beta = \pm\frac{k_{20}}{2\gamma} \tag{39}$$

representing the Eigen-value of the Eigen functions in (22). Returning now to (35) and analyzing the argument B_1, one can obtain that

$$B_1 = -k_{20}\gamma^{-1}\sqrt{\gamma^2 - \gamma^4 - \frac{1}{4}} = -ik_{20}\gamma^{-1}\sqrt{\left(\gamma^2 - \frac{1}{2}\right)^2} \tag{40}$$

where $\sqrt{\left(\gamma^2 - \frac{1}{2}\right)^2} \geq 0$ must be satisfied. As a result two cases are obtained, namely:

$$\gamma^2 - \gamma^4 - \frac{1}{4} > 0 \Leftrightarrow \gamma \neq \frac{1}{\sqrt{2}} \tag{41}$$

$$\gamma^2 - \gamma^4 - \frac{1}{4} = 0 \Leftrightarrow \gamma = \frac{1}{\sqrt{2}} \tag{42}$$

producing the stable spatial wave solutions of (22). Here, we are able to combine independent linear solutions of the $\sin(\beta_1 x)$ and $\cos(\beta_1 x)$. Using (35) together with (41) and (42) we then obtain Eigen-functions:

$$E(x,z) = \exp\left(-\frac{D_2 x}{2a_2}\right)\left\{A_1\cosh(B_2 x) + A_2\frac{\sinh(B_2 x)}{B_2}\right\} + g_2(x)\,\forall\gamma \neq \frac{1}{\sqrt{2}} \tag{43}$$

$$E(x,z) = \exp\left(-\frac{D_2 x}{2a_2}\right)\left\{A_1 + A_2 x\right\} + g_2(x)\,\forall\gamma = \frac{1}{\sqrt{2}} \tag{44}$$

for the x≥0 region, where $\mathrm{B}_2 = \sqrt{\left|\beta^2 - k_{20}^2\left(1-\gamma^2\right)\right|} = k_{20}\gamma^{-1}\sqrt{\left(\gamma^2 - 1\right)^2}$.

Now, we will obtain the conditions for the other medium. From Eq. (29), it follows that α_1, α_3 must be zero, and $\alpha_{01}^2C_1+\alpha_{01}D_1\varepsilon_1-\varepsilon_1^2 a_1=0$, $\alpha_{02}^2C_2+\alpha_{02}D_2\varepsilon_2-\varepsilon_2^2 a_2=0$. Then for x<0 the Eigen-functions are similar to (43) and (44) but with negative coefficient γ`. With the final value theorem in (29) we obtain:

$$F_2(\infty) = -\left\{\alpha_0\alpha_4^{-1}F_1(\infty) + \alpha_2\alpha_4^{-1}sG_1(\infty)\right\} \tag{45}$$

now using (30) we have:

$$G_2(\infty) = -\frac{\alpha_{02}F_2(\infty) + \varepsilon_1 G_1(\infty) + \alpha_{01}F_1(\infty)}{\varepsilon_2} \tag{46}$$

B. Eigen-mode solution for IM-PRC and OLD-PRC.

Now for IM-PRC and OLD-PRC, where $\gamma`=0$ and from (21), then continuing with the previous procedure and taking into account that, $\alpha_{01}=2n_1^2\gamma`/k_{10}= n_1^2D_1=0$ (see Appendix A), then $\alpha_3=$ and α_2 are modified as $\alpha_3=(a_2\varepsilon_2)^{-1}C_2\alpha_{02}\varepsilon_2$, $\alpha_2=(a_2\varepsilon_2)^{-1}ib_2\alpha_{02}\varepsilon_1$ and as $\alpha_5=0$ then we know that $\alpha_3=0$, which implies that $C_2=0$, solving $\beta=\pm k_{20}$ for our case $\beta=k_{20}$, substituting, now in (34) we obtain $B_1(s\to 0)=-ik_{20}\gamma$ for PRC medium in $x\geq 0$. For the surface wave formation the condition $B_1\neq 0$ must be fulfilled, and the surface wave solution is obtained from (35) as:

$$E(x)=\exp(-\frac{D_2x}{2a_2})\left\{A_1(z\to\infty)\cosh(k_{20}\gamma x)+A_2(z\to\infty)\frac{\sin(k_{20}\gamma x)}{k_{20}\gamma}\right\} \tag{47}$$

Where $A_1=G_2(s)$ and $A_2=D_2a_2^{-1}G_2(s)+2F_2(s)$, we take the limit and from (29), (30):

$$F_2(\infty)=-\left\{\frac{\alpha_0}{\alpha_4}F_1(\infty)-\frac{\alpha_0}{\alpha_4}F_1(0)\right\} \tag{48}$$

And

$$G_2(\infty)=-\frac{\alpha_{02}F_2(\infty)}{\varepsilon_2} \tag{49}$$

For x<0 regions the ideal metal:

$$E_{IM}(x)=0 \tag{50}$$

and for the optically linear medium:

$$E_{OLD}(x)=A\exp(-kx) \tag*{(50)}$$

VI. - SURFACE WAVE AMPLITUDE

A. Anti-symmetrical case

The following analysis is made for the case PRC`-PRC with the opposite sign of the nonlinearity diffusion. Considering the case $B_1=0$, (36), (45), (46), and the restriction according to that $g_2(x)=0$, and now considering (36) and as condition in this equation $A_1=0$, then we have**:**

$$E(x)=A_2\exp(-\frac{D_2x}{2a_2}) \tag{52}$$

Now it is necessary to find the constant A_2, considering A_1 in (46) we have:

$$A_1=\alpha_{02}F_2\left(\infty\right)-\varepsilon_1G_1(\infty)-\alpha_{01}F_1(\infty)=0 \tag{53}$$

$$A_2=\frac{D_2A_1(\infty)}{a_2}+2F_2(\infty) \tag{54}$$

then from (29), (30), and taking the limits for z→∞ and z→0 and solving such simultaneously, we obtain:

$$F_2(\infty) = -\left\{\frac{\alpha_0}{\alpha_4}F_1(\infty) + \frac{\alpha_2}{\alpha_4}G_1(\infty) - \frac{\alpha_0}{\alpha_4}F_1(0) - \frac{\alpha_2}{\alpha_4}G_1(0)\right\} \tag{55}$$

$$F_2(0) = -\left\{\frac{\alpha_0}{\alpha_4}F_1(0) + \frac{\alpha_2}{\alpha_4}G_1(0) - \frac{\alpha_0}{\alpha_4}F_1(0) - \frac{\alpha_2}{\alpha_4}G_1(0)\right\} = 0 \tag{56}$$

$$G_2(\infty) = -\frac{\alpha_{02}F_2(\infty) - \varepsilon_1 G_1(\infty) - \alpha_{01}F_1(\infty)}{\varepsilon_2} \tag{57}$$

$$G_2(0) = -\frac{\alpha_{02}F_2(0) - \varepsilon_1 G_1(0) - \alpha_{01}F_1(0)}{\varepsilon_2} \tag{58}$$

from (45) and (46), eliminating $F_2(\infty)$, we obtain:

$$G_1(\infty) = -\frac{a_1(\alpha_0 F_1(0) - \alpha_{01}G_1(0))}{ib_1\alpha_{01}} \tag{59}$$

To find the value of x_0 (corresponding to the maximum or minimum of the E(x) function, we calculate the derivative. This derivative equals to zero for:

$$x_0 = 2a_2 D_2^{-1} = 1/\gamma k_{20}$$

(the same result found in [5], but for another crystal orientation and oscillating guided waves, then we found that there is no evidence that the guided oscillatory waves are produced for both light polarizations). For $x \geq 0$ and for the other side, E(x) has a minimum that is localized in the point $x = x_0' = -2a_1 D_1^{-1} = 1/\gamma k_{10}$ (in [5] we used $x_0 = \gamma/|\Delta k|$ for this anti-symmetrical surface wave with another crystal orientation, see conclusions).

Calculating (52) with this value and considering that the $E(x_0)$ amplitude is normalized to the unit, and using (45) and (46) we have:

$$F_2(\infty) = \frac{D_2 \exp(1)}{4a_2} = \frac{\exp(1)}{2x_0} \tag{60}$$

See figure 3.

For the x<0 region, we have:

$$E(x) = \frac{x}{2x_0}\exp(1 - \gamma k_0 x) = \frac{1}{2}\gamma k_0 x \exp(1 - \gamma k_0 x) = \frac{k_0 x}{2\sqrt{2}}\exp\left(1 - \frac{k_0 x}{\sqrt{2}}\right) \tag{61}$$

The surface wave amplitude and argument depends on the diffusion photorefractive non-linearity.

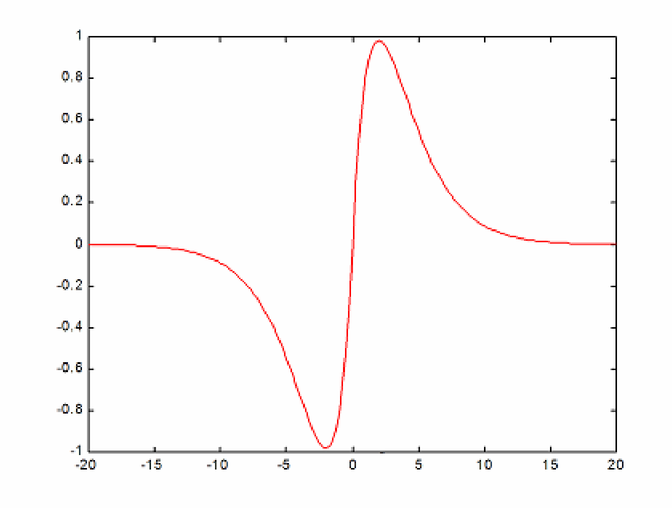

Figure 3. Surface wave propagation in PRC`-PRC media, for x<0 is $E_{PRC`} = \frac{ex}{2x_0}\exp(\frac{D_2 x}{2})$ and for x≥0 is $E_{PRC} = \frac{ex}{2x_0}\exp(-\frac{D_2 x}{2})$ and with $\gamma = \frac{1}{\sqrt{2}}$.

VII. - CONCLUSIONS

In this paper the dark conductivity and the saturation of the impurity photorefractive centers in the crystals are neglected, but this model could fundamentally offer an initial approximation. In principle, this solution can be generalized to take them into account.

The amplitude of surface waves determines how energy is transformed and what conditions are necessary for beam self-confinement.

From Eq. (61), it can be concluded that the maximum or minimum of the surface wave may be close to or coincide with the interface of the two media. Surface waves are formed due to the steady-state of the photoconductivity on the interface media (for PRC`-PRC, OLD-PRC media), and to the diffusion photorefractive non-linearity, diffraction (and internal reflection for OLD-PRC media). As shown in Eqs. (41) and (42), for this case β is determined as $\beta=\pm 2^{-1}\gamma^{-1}k_{20}$ for PRC`-PRC and $\beta=k_0$ for OLD-PRC and IM-PRC.

APPENDIX A:

Coefficients relating to the parameters of the two media:

$$\alpha_0 = \frac{ib_2\alpha_{02}\alpha_{01}}{a_2\varepsilon_2} \tag{A.1}$$

$$\alpha_1 = \frac{\alpha_{01}D_1}{a_1} + \frac{\alpha_{02}\alpha_{01}C_2}{a_2\varepsilon_2} - \varepsilon_1 \tag{A.2}$$

$$\alpha_2 = \frac{ib_2\alpha_{02}\varepsilon_1}{a_2\varepsilon_2} - \frac{ib_1\alpha_{01}}{a_1} \tag{A.3}$$

$$\alpha_3 = \frac{C_2\alpha_{02}\varepsilon_1}{a_2\varepsilon_2} - \frac{C_1\alpha_{01}}{a_1} \tag{A.4}$$

$$\alpha_4 = -\frac{ib_2\alpha_{02}^2}{a_2\varepsilon_2} \tag{A.5}$$

$$\alpha_5 = -\frac{\alpha_{02}^2 C_2 + \alpha_{02}D_2\varepsilon_2 - \varepsilon_2^2 a_2}{a_2\varepsilon_2} \tag{A.6}$$

$$\alpha_{02} = 2n_2^4 r_2 k_B T / e = 2n_2^2\gamma k_{20}^{-1} \tag{A.7}$$

$$\alpha_{01} = 2n_1^4 r_1 k_B T / e = 2n_1^2\gamma k_{10}^{-1} \tag{A.8}$$

Where ε_1, ε_2 are the dielectric constants of each medium, $a_1=k_{10}^{-2}$, $a_2= k_{20}^{-2}$, $b_1=2\beta_1 a_1^{-1}$, $b_2=2\beta_2 a_2^{-1}$, $C_1=(\beta_1^2 - k_{10}^2)\, k_{10}^{-2}$, $C_2=(\beta_2^2 - k_{20}^2)\, k_{20}^{-2}$.

REFERENCES

[1] G. Duree, J. Shultz, G. J. Salamo, M. Segev, A. Yariv, B. Crosignani, P. Di Porto, E. J. Sharp, and R. R. Neurgaonkar, Phys. Review letter **71**, 533 (1993).

[2] M. D. Iturbe Castillo, P. A. Marquez Aguilar, J. J. Sanchez Mondragon, S. Stepanov, and V. Vysloukh, Appl. Physics Letters **64**, 1 (1994).

[3] M. Segev, B. Crosignani, A. Yariv, and B. Fisher, Physical Review Letters **68**, 923 (1992).

[4] B. Crosignani, M. Segev, D. Engin, P. Di Porto, A. Yariv, and G. Salamo, J. Opt. Soc. Am. B. **10**, 446 (1993).

[5] G.S. Garcia Quirino, J.J. Sanchez Mondragon, and S.Stepanov, Phys. Rev. A **51**, 1571 (1995).

[6] G.S. Garcia Quirino, J.J. Sanchez Mondragon, and S.Stepanov, V.A. Vysloukh. J. Opt. Soc. Am. B. **13** 2530 (1996).

[7] M.Cronin-Golomb "Photorefractive surface waves," Opt. Lett. **20**, 2075 (1995).

[8] A.V. Khomenko, Ervin Nippolainen, Alexei A. Kamshilin, A. Zuniga Segundo, Timo Jaaskelainen., Leaky photorefractive surface waves in $Bi_{12}TiO_{20}$ and $Bi_{12}SiO_{20}$ crystals, Optics Communications **150**,175 (1998).

[9] A.V. Khomenko, A. Garcia-Weidner and A.A. Kamshilin, Amplification of optical signals in $Bi_{12}TiO_{20}$ crystal by photorefractive surface waves, Optics Letters **21**,1014 (1996).

[10] Jackson Classical Electrodynamics, second edition page 303-306, **QC631.J3**.

[11] F. P: Strohkendl, J. M. C. Jonathan, and R. W: Hellwarth. Optics letters **11**, 312 (1986).

[12] M.P. Petrov, S.I. Stepanov, and A.V. Khomenko, Photorefractive Crystals in Coherent Optics (Springer, Berlin, 1991).

[13] Physics of semiconductors devices S.\ M. Sze. Wiley International Edition 1969

[14] J.J. Sakurai Modern Quantum Mechanics Page 110-112, **QC174.12.S25**.