REVISTA COLOMBIANA DE FÍSICA, VOL.38, No.1, 2006

THE TWO-PHOTONS TRANSITION OF 2S-1S STATES IN HYDROGEN ATOMS AND ITS ASTROPHYSICAL APPLICATION

Camilo Mejía Prada[1]*,Benjamin Calvo Mozo*[2]
[1]*Departamento de Física,Universidad Nacional de Colombia*
[2]*Observatorio Astrónomico Nacional, Universidad Nacional de Colombia*
(Recibido 18 de Sep.2005; Aceptado 05 de Feb.2006; Publicado 05 de Abr.2006)

RESUMEN

En el presente artículo consideramos la técnica de los estados virtuales para calcular la probabilidad de la transición 2S-1S bifotonica en el átomo de hidrógeno y comparar el flujo así obtenido con el continuo en el azul de una muestra de objetos Herbig-Haro, mostrando una excelente coinsidencia.

Palabras claves: Hidrógeno: Continuo bifotónico 2S-1S, objetos Herbig-Haro.

ABSTRACT

Present work deals with the virtual state tecnique to account for the 2S-1S two photons continuum emission of the hydrogen atom and compare the obtained flux with the blue bump observed in a sample of Herbig-Haro object in good agreement.

Keywords: Hydrogen: 2S-1S two photons continuum Herbig-Haro object.

Introduction

Hydrogen is the most important element in the universe; in particle number for typical nebulae is on order on the ninety percent of all atom present there. So when a central source emits photons with energies greater or equal than 13,6eV the cloud becomes ionized. Typical examples are HII regions, planetary nebulae, and AGNs. On normal conditions of photoionized nebulae almost 32 % of the recombination of hydrogen atoms fall down to the 2^2S level. The transition from this state to the ground state is strictly forbidden. Present work describes the probability for this transition by means of a two photons transition, the virtual state approach. We compare the probability spectrum with the bump found in the continuum blue region of some HH-objects.

The two photon process.

We can use the non-relativistic Hamiltonian for an electron interacting with an electromagnetic field,which can be splitted in three terms $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2$, where $\mathcal{H}_0 \equiv \frac{P^2}{2m} - e\phi$, ϕ the coulomb potencial, $\mathcal{H}_1 \equiv \frac{e}{2mc}\left[\mathbf{P}\,.\,\mathbf{A} + \mathbf{A}\,.\,\mathbf{P}\right] = \frac{e}{mc}\mathbf{A}\,.\,\mathbf{P}$, is the photon-electron interacting term (Coulomb's gauge) and $\mathcal{H}_2 \equiv \frac{e^2}{2mc^2}\mathbf{A}\,.\,\mathbf{A}$, is the hamiltonian part that involve two photons.

For the hydrogen atom $\mathcal{H}_0$ describes its stationary structure while $\mathcal{H}_1$ describes the electron-photon interaction leading to radiative transitions in the atom. Usually, the density of photons in astrophysical context is by many orders of magnitude much lesser that $n_\gamma \sim 10^{24}cm^{-3}$; this fact enables us to treat $\mathcal{H}_1$ as a perturbation, so we can expand the state function $\psi(\mathbf{r}, t)$ of the $\mathcal{H}_0 + \mathcal{H}_1$ system in terms of the complete set of eigenfunctions of $\mathcal{H}_0$:

$$\psi(\mathbf{r}, t) = \sum_j c_j(t)\varphi_j(\mathbf{r})e^{-i\mathcal{E}_j t/\hbar} \tag{1}$$

where $\langle\psi_a|\psi_b\rangle = \delta_{ab}$, for bound states.

If we expand $\mathbf{A}$ of the photon field in a cavity of volume V we have:

$$\mathcal{H}_1 = \frac{e}{mc}\mathbf{A}\,.\,\mathbf{P} = \sum_{\mathbf{k},\alpha}[\mathcal{H}_\alpha^{abs}(\mathbf{k})e^{-i\omega t} + \mathcal{H}_\alpha^{em}(\mathbf{k})e^{i\omega t}], \tag{2}$$

where:

$$\mathcal{H}_\alpha^{abs}(\mathbf{k}) = \frac{e}{m}\left[\frac{hN_{k,\alpha}}{V\omega}\right]^{1/2} e^{i\mathbf{k}.\mathbf{r}}\mathbf{e}_\alpha(\widehat{\mathbf{k}})\,.\,\mathbf{p}, \tag{3}$$

is the Hamiltonian associated to photons absorption with wave vector $\mathbf{k}$ and polarization state α, and

$$\mathcal{H}_\alpha^{em}(\mathbf{k}) = \frac{e}{m}\left[\frac{h(N_{k,\alpha}+1)}{V\omega}\right]^{1/2} e^{-i\mathbf{k}.\mathbf{r}}\mathbf{e}^*_\alpha(\widehat{\mathbf{k}})\,.\,\mathbf{p} \tag{4}$$

is the Hamiltonian associated to photon emission.
Hereafter we are interested in the Hamiltonian eq.(4), for we are considering the emission of photons, associated to a transition from some bounded initial state $|i\rangle$ of the hydrogen atom to some final $|f\rangle$ one.
If we apply the expansion eq.(1) to the hamiltonian $\mathcal{H}_0+\mathcal{H}_1$ and make the inner product with some bra $\langle\psi_f|$, taking into account the orto-normality of the eigenkets of $\mathcal{H}_0$, we find the coefficients of the expansion eq.(1). The sum of the square absolute values of them over all wave vectors $\mathbf{k}$, polarization α and space directions gives us the probability of the transition:

$$\mathcal{P}_{if} = \left(\frac{e}{2\pi m}\right)^2\sum_{\alpha=1}^{2}\int\frac{N_{k,\alpha}}{\hbar\omega}|\langle\varphi_f|e^{-i\mathbf{k}.\mathbf{r}}\mathbf{e}_\alpha(\widehat{\mathbf{k}})\,.\,\mathbf{p}|\varphi_i\rangle|^2 \frac{\sin^2[(\omega_{fi}+\omega)t/2]}{[(\omega_{fi}+\omega)/2]^2}\frac{\omega^2}{c^3}d\omega d\Omega \tag{5}$$

It is customary to use the approximation $\frac{\omega_{fi}t}{2} \gg 1$, which gives:

$$\mathcal{P}_{if} = t\left(\frac{e^2}{hc^3m^2}\right)\sum_{\alpha=1}^{2}\oint\left[\omega N_{k\alpha}|\langle\varphi_f|e^{-i\mathbf{k}.\mathbf{r}}\mathbf{e}_\alpha(\widehat{\mathbf{k}})\,.\,\mathbf{p}|\varphi_i\rangle|^2\right]_{fi}d\Omega \tag{6}$$

Thus, we arrive to the Fermi golden rule

$$\frac{\partial\mathcal{P}_{if}}{\partial t} = \sum_{\alpha=1}^{2}\oint\omega_\alpha d\Omega, \tag{7}$$

where

$$\omega_\alpha = \frac{2\pi}{\hbar}|\langle\varphi_f|\mathcal{H}_\alpha^{int}(\widehat{\mathbf{k}})|\varphi_i\rangle|^2\rho_{\alpha\omega}(\widehat{\mathbf{k}}) \tag{8}$$

If we consider the wave function of the 2S-1S states of the hydrogen atom, and take into account eq.(3),(7),(8), we conclude that the 2S-1S transition is completely forbidden for the a single photon radiative transition. So we shall consider now the 2S-1S transition by means of the emission of two photons. To carry out the calculation of the transition probability we deal with the so called virtual level method.
In this method one employs some virtual level (see fig.1), such that $\hbar(\omega_1+\omega_2) = \mathcal{E}_{2S}-\mathcal{E}_{1S}$.

In this approach $l = 1$ for the virtual state, so one can use the description for dipolar transitions $e^{-i\mathbf{k}\cdot\mathbf{r}} \approx 1$ in the composition of "transitions" $2S \rightarrow np$, followed by $np \rightarrow 1S$, where np denotes the virtual level. However, the electron never jump to the virtual level;it is only a way to carry out our calculations.

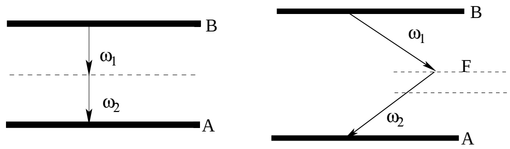

Figura 1: *Two-photon emission.*

Thus, we take the vector potencial $\mathbf{A}$ in $\mathcal{H}_{int} = \frac{e}{mc}\mathbf{A}\cdot\mathbf{p}$, as the sum of the two potentials associated to each photon,take into account eqs.(8),(7), and putting $\Gamma_{ab} = \frac{\partial \mathcal{P}}{\partial t}$, to arrives to:

$$\Gamma_{ab}(\omega_1) = \frac{\pi E_1^2 E_2^2 e^4}{8\hbar^4}\left|\sum_f \left[\frac{\langle b|\mathbf{r}\cdot\widehat{\mathbf{e}}_2|f\rangle\langle f|\mathbf{r}\cdot\widehat{\mathbf{e}}_1|a\rangle}{\omega_1+\omega_{fa}} + \frac{\langle b|\mathbf{r}\cdot\widehat{\mathbf{e}}_1|f\rangle\langle f|\mathbf{r}\cdot\widehat{\mathbf{e}}_2|a\rangle}{\omega_2+\omega_{fa}}\right]\right|^2 \times \delta(\omega_{ba}+\omega_1+\omega_2) \tag{9}$$

where kets $|a\rangle, |b\rangle, |f\rangle$ are initial, final and virtual states, respectively.
As we are dealing with a continuum due to photons emission, we consider the number of photons in the frequency interval $(\omega, \omega + d\omega)$,thus,to calculate the Einstein coefficient for the hydrogen 2S-1S two-photon transition, $A_{2S,1S}$, we integrate eq.(9) with respect to all directions, frequencies and consider the description of continuum states given by Sttobe.
Doing so, one obtains

$$A_{2S,1S} = \frac{9\alpha^6 cR}{2^{11}}\int_0^1 G(y)dy \tag{10}$$

where α is the fine structure's constant, R is Rydberg's constant, $y = \nu_1/\nu_{(L_{y\alpha})}$, $\nu_2 = (1-y)\nu_{(L_{y\alpha})}$,$\nu_1, \nu_2$ are the frequencies of the two emitted photons, $\nu_{(L_{y\alpha})}$ in the frequency of a Lymann α photon, and

$$G(y) = y^3(1-y)^3\left|\sum_{m=2}^{\infty} R_{mp}^{1S}R_{mp}^{2S}\left(\frac{3}{1+3y-4/m^2}+\frac{3}{4-3y-4/m^2}\right) + \int_0^{\infty} C_{1S}C_{2S}dx\left(\frac{3}{1+3y+4x^2}+\frac{3}{4-3y+4x^2}\right)\right|^2. \tag{11}$$

In the above expression x denotes the real number associated to a continuum state with energy $E = x^2\frac{e^2}{2a_0}$, where $a_0 = \frac{\hbar^2}{me^2}$.

Astrophysical Application

A numerical integration in function $G(y)$ gives $\int_0^1 G(y)dy = 3{,}77$ which after replacing into eq.(10) gives the total transition probability $A_{2S,1S} = 8{,}227seg^{-1}$. However, this transition probability takes into account all photons emitted in the range of frequencies $0 < \nu < \nu_{(L_{y\alpha})}$ the probability to have a photon of frequency ν in that range.

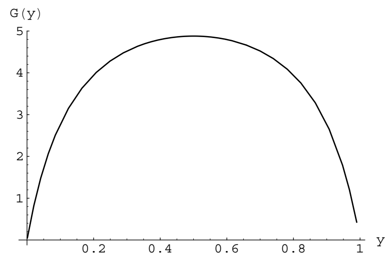

Figura 2: *Probability of two-photon emission.*

However, the specific emissivity per gram of material, j_ν, is directly proportional to $n(2S)yG(y)$, where $n(2S)$ is the density of states $2S$ of hydrogen atoms.
Further, the flux $F_\nu \propto j_\nu$, $|d\nu| = c\lambda^{-2}d\lambda$, and $F_\nu d\nu = F_\lambda d\lambda$. So we have $F_\lambda = (F_\nu)|_\lambda c\lambda^{-2}$, where $(F_\nu)|_\lambda$ stands for the flux F_ν replacing $\nu = c/\lambda$.
We shall consider now the "blue" region transmitted by our atmosphere, say, in the range 3000$\AA$-4000$\AA$ in order to compare the continuum of some astrophysical sources in that spectral region with our calculated F_λ, To carry it out we take a sample of points of $G(y)$ and approaches it by a power law $G(y) \propto y^n$, which gives $n \simeq 0{,}2$, which in turns implies $F_\lambda \propto \lambda^{-3,2}$. We confront this result with the blue part of the spectra of a sample of ten HERBIG-HARO objects [5] they reported a power law continuum, $F_\lambda \propto \lambda^{-3,3}$ in the same spectral region.

Discussion

In present work we presented a brief account of the biphotonic trasition 2S-1S for hydrogen atoms. It is possible to observe it only in astrophysical photoionised nebulae, for their low densities enable successfully this process. This transition implies a blue bump in the spectra of many photoionised nebulae. We compare our theorical result with the blue part of spectra of HERBIG-HARO objects, given good agreement with observational results. However, the method developet in present work and applied to the reverse problems is an interesting spin-off which can find applications in multi-photon process problems [6].

Bibliography

[1] Shu. F. H, The physics of Astrophysics,1991,vol 1 Radiation, University Science, Mill Valley.

[2] Stobbe, Ann.d.Phys.,1930,**7**,661.

[3] Teller, and Breit, Ap.J.,1940,**91**,215.

[4] Spitzer, L,Jr, and Greenstein J.L., Ap.J,1951,**114**,407.

[5] Dopita, M.A.,Binette,L. and Schwarts, R.D.,Ap.J.,1982,**261**,183.

[6] Kimdliya, R., Prasad, V. and Mohan, M.,J. Phys. B; At.Mol.Opt.Phys.,2000,**33**,5263-5273.