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DOPPLER EFFECT OF A SOUND SOURCE MOVING WITH SIMPLE HARMONIC MOTION

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RESUMEN

Se presenta una experiencia demostrativa que permite cuantificar el corrimiento Doppler de la frecuencia emitida por una fuente sonora que oscila armónicamente con relación a un observador en reposo. La fuente se acopla a un oscilador masa-resorte, y se somete a un movimiento oscilatorio con velocidades mucho menores que la velocidad del sonido en el medio, esto origina un desplazamiento armónico en la frecuencia observada por un receptor en reposo. El corrimiento Doppler se visualiza mediante un espectrograma, permitiendo confrontar el modelo teórico con el experimento.

Palabras claves: Efecto Doppler, espectrograma Doppler, movimiento armónico simple

ABSTRACT

Doppler effect of a source moving with simple harmonic motion is presented. The emphasis is placed when the source oscillates with speed lower than the speed of sound in the medium. The temporal evolution of the observed frequency by a receiver staying in rest is displayed in a spectrogram. Mathematical considerations and experimental results are given. The basic setup for classroom experiences is shown.

Keywords: Doppler effect, Doppler spectrogram, simple harmonic motion

1. Introduction

Acoustic Doppler effect is a phenomenon very well-known [1]. It takes place due to the relative motion between an observer and a sound source. Doppler effect, also called Doppler shift, is characterized by the change of the observed frequency with respect to the frequency emitted by the sound source.

A demonstrative experience allowing the quantitative determination of the Doppler shift of the frequency emitted by a transmitter moving with simple harmonic motion is presented. Generally the basic setup for quantitative demonstrations of the Doppler effect requires ultrasonic transducers in the emission and reception of the signal, however in this work an audible sound wave is employed. Some experiences of didactic character to illustrate the Doppler shift, with relative low cost and easy implementation have been recently proposed using transducers in this range of frequencies [2,3].

In this work initially the theoretical framework is considered, then the experimental setup is described, some basic aspects of a Doppler spectrogram are given and finally the results and conclusions are shown.

2. Theoretical Framework

A receiver in rest, relative to the medium of propagation of a sound wave, observes in its reference frame, a higher frequency than the frequency emitted by a source when this comes closer toward it. The frequency observed by receiver is smaller when the source moves away from it.

A receiver in front of source observes a shortening in the wavelength, that is, an increment in the measured frequency in comparison with the frequency of the source. A receiver, behind of the source observes a longest wavelength or decreasing the frequency.

The observed frequency in the reference frame of the receiver, $\mathrm{f_o}$, when the source moves on the line that joint them, with a speed, $\mathrm{v_s}$, is given by the equation

$$\mathrm{f_o} = \frac{\mathrm{f_s}}{\left(1 \pm \frac{\mathrm{v_s}}{\mathrm{u}}\right)} \tag{1}$$

Where, $\mathrm{f_s}$ is the frequency in the reference frame of the source and u is the speed of the sound in the medium [1]. The sign (+) in the equation (1) is used when the transmitter moves away from the receiver and the sign (-) when the transmitter moves toward the receiver.

If the source presents a simple harmonic motion, with angular frequency ω, amplitude, A and constant of phase φ, its speed vary according to,

$$\mathrm{v_s}(\mathrm{t}) = \omega A \cos(\omega t + \varphi) \tag{2}$$

With (2), the equation (1) can be written

$$\mathrm{f_o} = \mathrm{f_s}\left(1 + \frac{\omega \mathrm{A} \cos(\omega t + \varphi)}{\mathrm{u}}\right)^{-1} \tag{3}$$

When the maximum speed reached by the sound source is smaller that the speed of the sound in the medium, the expression (3) can be developed in series expansion. Taking the two first terms, the observed frequency become

$$\mathrm{f_o} \approx \mathrm{f_s}\left(1 - \frac{\omega \mathrm{A} \cos(\omega t + \varphi)}{\mathrm{u}}\right) \qquad \text{with} \quad \omega A << \mathrm{u} \tag{4}$$

With this approximation, Doppler shift $\Delta \mathrm{f}$ results harmonic and proportional to the real frequency emitted by the sound source.

$$\Delta \mathrm{f} = \mathrm{f_o} - \mathrm{f_s} \approx -\mathrm{f_s}\frac{\omega \mathrm{A} \cos(\omega t + \varphi)}{\mathrm{u}} \tag{5}$$

3. Experimental Setup

The sound source employed in the experiment consists of a piezoelectric tweeter, with a range of frequencies between 5 [kHz] - 20 [kHz]. The tweeter is connected to a wave generator (Also PC sound card and suitable software could be used), see figure 1. The source is attached to a spring with a force constant, $k = 1.45 \pm 0.02$ [kg/s^2]. The mass oscillating is $m =$ 45.0 ± 0.1 [g]. The spring-mass system gives to the source, respect to receiver, a simple harmonic motion with period T =1.10 [s]. Having a motion amplitude ωA=0.07± 0.01[m] and an angular frequency $\omega = 2\pi/T$, the maximum speed reached by the source, ωA results 0.40[m/s]. Considering the sound speed 346[m/s], the condition $\omega A <<$ u , is satisfied.

The sound is recorded by a sound card and sampled at frequency 44100 [Hz], to 16 bits/sample. The frequency determination is performed numerically using Matlab$^{®}$ software.

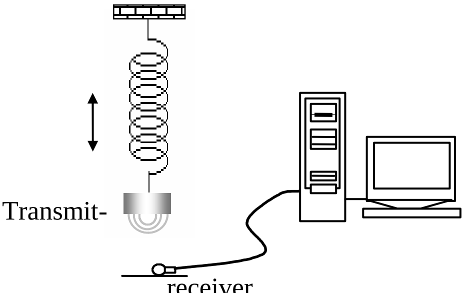


Figure 1. Experimental setup used to determine the Doppler shift of a sound source moving with simple harmonic motion.

4. Spectrogram

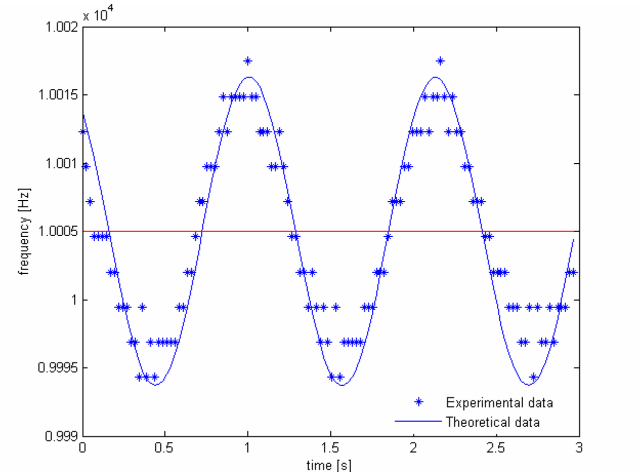
Spectrogram or time-frequency diagram results useful to visualize the temporary evolution of the Doppler shift of the frequency emitted by a sound source when exhibit a harmonic motion. Spectrogram is a matrix; its columns are formed with the magnitude of discrete Fourier transforms of sound segments. These segments constitute vectors that could be overlapped and on which the signal is approximately stationary. Eventually in a too long segment the signal could change significantly, by contrary, a too short segment could not be representative for the signal.

5. Results

In the experiment the frequency emitted by the source was established in 10005 [Hz]. The temporary evolution of the frequency due to the oscillations of the source it is observed in the spectrogram in figure (2). The sound segment has 1024 samples, corresponding approximately to 24.3 [ms]. With the aim to increase the precision in the Doppler shift determination, the vector of the sound segment is zero padded until complete 16384 samples and multiplied by a Hanning window of the same size and then Fourier transformed.

Spectrogram, in figure (2) shows the theoretical and experimental data corresponding to the Doppler shift of the maximum of the Fourier transform during the first 3 [s] of motion. The constant of phase in equation (2) is adjusted to $5\pi/7$ [rad].

Figure 2. Spectrogram showing theoretical and experimental data of the Doppler shift.



6. Conclusions

A demonstrative experience has been presented allowing quantifying the Doppler shift of a sound emitted by a source moving with simple harmonic motion.

The coincidence between the theoretical and experimental data take out of a spectrogram allows validating the experience.

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