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SPECTRUM OF TWO SPATIALLY SEPARATED PARTICLES IN COAXIAL NANORINGS

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ABSTRACT

Exact solution for two spatially separated particles in two coaxial one-dimensional nanorings has been obtained by using trigonometric sweep method. The structure of the energy spectrum of two electrons and electron-hole pair in nanoring with different sizes has been calculated. The comparative analysis of the quantum size effect on the energy levels in the case of the electron-hole attraction and the electron-electron repulsion is presented. As the radii of coaxial rings merge our results coincide with those obtained previously for one-dimensional nanoring.

Key Words: trigonometric sweep, nanoring, electron-hole, electron-electron

Introduction

In the last few years, self-assembled quantum dots (QDs) with different morphologies have been experimentally fabricated and theoretically studied. These nanostructures can be used for high-performance device applications as well as for exploration of novel physical properties associated with the strong competition between the QD´s sizes, Coulomb interaction and magnetic confinement. Recently, Lorke and co-workers reported the fabrication of InGaAs QDs with a ring-like geometry containing two particles [1] in which could be observed such phenomena as Aharonov–Bohm effect, persistent currents and quantum chaos. Stimulated by this finding some theoretical investigations about one- and two-particle spectra in quantum rings (QRs) have been done to reveal the peculiarities related to the ring-like geometry [2]. One of the possible realistic physical situations is a two-electron InGaAs QR with a very large outer radius (~400-500nm) and a very small width (~20-40nm) and thickness (~2-4nm). In these conditions the adiabatic approximation allows one to decouple the slow rotation along the ring from the rapid transverse oscillations in the radial and azimuthal directions and to reduce the analysis of the low-lying energy levels to a one-dimensional eigenvalue problem leaving to aside the motion in radial and *z*-directions. Exact energy levels of two-electron systems in 1D QRs have been obtained and the size effect on the interaction energies in the limit of a narrow-width nanoring have been studied in Ref. [3]. Only a few two-particle exactly solvable problems are known and therefore the exact solution found out in Ref. [3] is interesting not only from a physical point of view but also from a mathematical point of view. On the other hand, one could suppose that this exact solution found only for two identical particles might be extended on the case of two different particles, for example for the electron-hole pair replacing only the Coulomb repulsion by the attraction. But it is well-known, that any one-dimensional two-particle system with Coulomb attraction is instable with respect to the fusion and it is a reason why we propose to consider in this work a model in which two particles are located in two different concentric 1D rings with a small separation. The exact solution for this model can

be also obtained and a difference between the cases of repulsion and attraction can be analyzed within framework of the same model.

Theory

Although the exact solution can be also obtained for two particles with different masses we intentionally consider below a simplified model of two particles with the same effective mass m^* in order to have the possibility to compare our results with those from Ref. [3]. As well as in Ref. [3] the effective Bohr radius, $a_0^* = \hbar^2 \varepsilon / m^* e^2$ and the effective Rydberg $Ry^* = m^* e^4 / 2\hbar^2 \varepsilon^2$ we use as the length and energy units whose typical values for InGaAs materials are of the order 10 nm and, 5meV respectively. In our model the first particle is located within the inner ring of the radius $R_1 = \beta R$; $0 < \beta \le 1$ and other particle is located within the outer concentric ring of the radius $R_2 = R$. When the ratio of inner to outer radii tends to one ($\beta \to 1$) our results for two-electron system should coincide with those from Ref. [3]. Following to notations of Ref. [3] we represent the renormalized dimensionless 1D Hamiltonian, $\tilde{H}$ for two particles in polar coordinates as:

$$\tilde{H} = HR^2 = -\frac{1}{\beta^2}\frac{\partial^2}{\partial \varphi_1^2} - \frac{\partial^2}{\partial \varphi_2^2} + 2\tau R \Big/ \sqrt{(1-\beta)^2 + 4\beta\, sen^2\, \frac{\varphi_1 - \varphi_2}{2}}\,;\ \tau = \pm 1 \qquad (1)$$

The positive sign of the parameter τ in the Hamiltonian (1) corresponds to two-electron system and the negative sign to the electron-hole pair. The eigenvalues $\tilde{E}$ of the Hamiltonian (1) are the two-particle energies E multiplied by the squared radius of the outer ring ($\tilde{E} = ER^2$). One can find the eigenvalues, $\tilde{E}$ exactly by using center-of mass $\theta = (\varphi_1 + \varphi_2)/2$ and relative $\varphi = \varphi_1 - \varphi_2$ coordinates for which the Hamiltonian (1) is separated in two independent parts $\tilde{H}_c$ and $\tilde{H}_r$:

$$\tilde{H} = \tilde{H}_c + \tilde{H}_r;\quad \tilde{H}_c = -\frac{1}{1+\beta^2}\frac{\partial^2}{\partial \theta^2};\quad \tilde{H}_r = -\frac{1+\beta^2}{\beta^2}\frac{\partial^2}{\partial \varphi^2} + \frac{2\tau R}{\sqrt{(1-\beta)^2 + 4\beta\, sen^2\, \varphi/2}} \qquad (2)$$

This separability allows us to write the two-particle normalized energy in the form $\tilde{E} = \tilde{E}_c + \tilde{E}_r$, where $\tilde{E}_c$ and $\tilde{E}_r$ are the normalized energies of the center-of-mass and the relative rotations, which are eigenvalues of the operators $\tilde{H}_c$ and $\tilde{H}_r$, respectively. The eigenvalues of the operator $\tilde{H}_c$ are given by: $\tilde{E}_c(M) = M^2/(1+\beta^2)$ with center-of-mass angular momentum $M = 0, \pm 1, \pm 2, \ldots$, whereas the eigenvalues of the operator $\tilde{H}_r$ may be found only by solving numerically the one–dimensional wave equation:

$$-\frac{1+\beta^2}{\beta^2}\frac{\partial^2 f_{ms}(\varphi)}{\partial \varphi^2} + \frac{2\tau R}{\sqrt{(1-\beta)^2 + 4\beta\, sen^2\, \varphi/2}}\, f_{ms}(\varphi) = \tilde{E}_c(m,s) f_{ms}(\varphi) \qquad (3)$$

in the region of $[0, 2\pi]$ with periodic boundary conditions $f_{ms}(0) = (-1)^m f_{ms}(2\pi)$. The quantum number $m = 0, \pm 1, \pm 2, \ldots$ defines the two-particle relative angular momentum

and $s = \pm$ the parity of the solution of Eq. (3). In the case of two-electron system the even solutions $(s = +)$ correspond to singlet states and the odd solutions $(s = -)$ correspond to triplet states. The electronic spectrum of the two-particle system will then be composed by the set of states $\tilde{E}(M, m, s)$ given by three quantum numbers M, m, s.

Results

In order to obtain the solution of Eq. (3), we use the trigonometric sweep method [4]. We have performed numerical calculations for energy levels $\tilde{E}(M, m, \pm)$ of two electrons and electron-hole pair in two concentric 1D QRs with different radii R and ratios β. In order to check the accuracy of our numerical procedure, we first calculated the energies of the low-lying states of two electrons in concentric 1D QRs with some different close to one ratios inner to outer radii β as a function of the outer radius R and compared our results with those obtained previously in Ref. [3] for two electrons in 1D nanoring by using series expansion method. We found that our results consistently tend to those from Ref. [3] as the parameter β tends to one. One example of such tendency we show in Fig. 1 for low lying $(0,0,+)$ singlet state and for $(0,2,-)$ triplet state. The lines for $\beta = 1.0$ in this figure are from Ref. [3] and they correspond to the case when two rings merge forming only one ring.

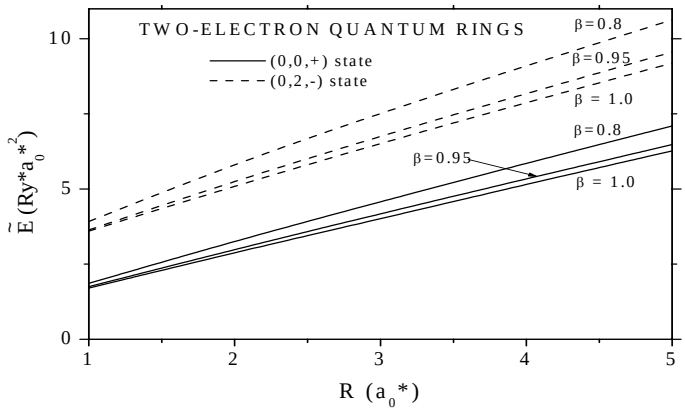

Fig. 1. Normalized energies $\tilde{E}(0,0,+)$ and $\tilde{E}(0,2,-)$ of two electrons in two coaxial QRs with different ratios inner to outer radii β as a function of the outer radius R. The lines for $\beta = 1.0$ are from Ref. [3] and for $\beta = 0.8$ and $\beta = 0.95$ are our results.

One can see from Fig. 1 that the energies of two-electron system in a par concentric QRs with inner radius βR and outer radius R always are greater than those in an alone QR of radius R. It is due to the fact that the largest possible separation between two electrons in concentric QRs, $(1+\beta)R$ is smaller than the corresponding value $2R$ in an alone QR and only when $\beta \to 1$ the separations in both systems become of the same order. In Fig.2 we present the radius dependency of the energies of the low-lying levels corresponding to even states for

the electron-hole pair (the left side in Fig.2) and two-electron system (the right side in Fig.2) located in concentric QRs with $\beta = 0.95$.

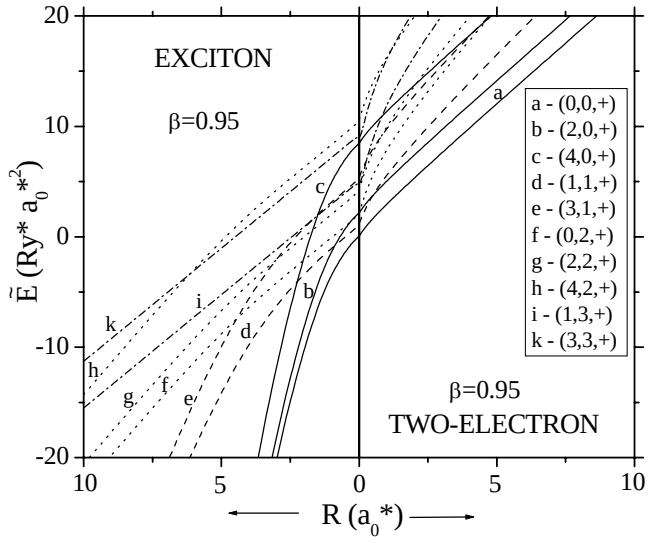

Fig. 2. Normalized energies of low-lying even states for the electron-hole pair (the left side) and two electrons (the right side) in coaxial QRs with $\beta = 0.95$ as a function of the outer radius

As the outer QR radius increases from zero to ten, the quantum-size effects modify the level-ordering and lead to crossover of some levels. It is seen from Fig.2 that the Coulomb interaction is relevant when the confinement is small $(R \to \infty)$ and the levels form new groups which are very different from the level groups typical for a strong confinement $(R \to 0)$. In the first case in Hamiltonian (2) the term of the Coulomb interaction is dominant and therefore the groups are formed by levels with the same values of the relative angular momentum m whereas in the second case the term of the Coulomb interaction in the Hamiltonian becomes depreciable and the exact normalized energies tend to the value $\left(M^2 + m^2\right)/2$.

In conclusion, we consider a new exactly solvable two-particle model which may be useful for further theoretical and experimental studies of narrow quantum rings.

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